Sets and Counting

- Sets and Set Operations (II.A.1,2)
- Number of Elements in a Finite Set (II.A.3)
- Multiplication Principle (II.A.4a,4b)
- Permutations and Combinations (II.A.4c,4d,4e)
Set Terminology

- A set is a well-defined collection of objects.
- The objects or members of a set are known as the elements of the set.
- There are two ways to write a set:
  - Roster Notation – all elements are listed
    - Like names on a roster.
      Example: \( F = \{1, 2, 3, 4\} \)
      Example: \( H = \{0, 1, 2, \ldots, 100\} \)
  - Set-Builder Notation – a description of the elements in the set is given
    - Decide whether or not an object fits the description.
Set Equality

- Two sets are *equal* if and only if they have exactly the same elements.
- Order of the listed elements does not matter!

*Example:* Which of the following sets are equal?

\[
A = \{a, b, c, d\} \quad B = \{a, c, d\} \quad C = \{d, c, a, b\}
\]

\[
R = \{x \mid x \text{ is a number evenly divisible by } 2\} \quad S = \{0, 2, 4, 6\}
\]
Subsets

- If every element of a set \(E\) is also an element of a set \(F\), then \(E\) is a subset of \(F\), denoted \(E \subseteq F\).

Example: Are any of the following sets subsets of one another?

\[A = \{a, b, c, d\} \quad B = \{a, c, d\} \quad C = \{d, c, a, b\}\]

\[R = \{x \mid x \text{ is a number evenly divisible by 2}\} \quad S = \{0, 2, 4, 6\}\]
Proper Subsets

The set \( E \) is a *proper subset* of the set \( F \), denoted \( E \subset F \), if

1. \( E \) is a subset of \( F \)
2. \( F \) contains more elements than \( E \)

*Example:* Are any of the following sets proper subsets of one another?

\[
A = \{a, b, c, d\} \quad B = \{a, c, d\} \quad C = \{d, c, a, b\} \\
R = \{x \mid x \text{ is a number divisible by 2}\} \quad S = \{0, 2, 4, 6\}
\]
The Empty Set

- The set which contains no elements is called the empty set.
- Denoted by EITHER \{\} OR \emptyset.
- The empty set is a subset of every set!
Example: Given the sets $A=\{0, 2\}$ and $B=\{e, f, g\}$, find and list all subsets of $A$ and $B$.

- In general, a given set will have how many
  - subsets?
  - proper subsets?
The Universal Set

- A universal set, \( U \), is the set of all elements of interest in a particular problem.

*Example:* What universal set would make sense in the following situations?

a) You are asked to find the ratio of male shoppers to female shoppers in the local mall at a specific time.

b) You are asked to find the ratio of male shoppers to female shoppers in a particular store of the local mall at a specific time.
A *Venn diagram* can be used to visually represent sets and the relationships between them.

- The universal set, $U$, for a problem will be represented by a rectangle.
- Subsets of $U$ will be represented by regions inside the rectangle.
**Example:** Idea University is located in Smartville. Let

\[ U = \{ x \mid x \text{ is a person living in Smartville} \} \]

\[ A = \{ x \mid x \text{ is a person living in Smartville and enrolled at Idea University} \} \]

\[ B = \{ x \mid x \text{ is a person living in Smartville and enrolled in a math class at Idea University} \} \]

Draw a Venn diagram representing the relationship between these sets.
Example: In the Venn diagram below, all known elements of the sets are placed within the diagram. Write the sets listed.

\[ U = \]

\[ A = \]

\[ B = \]

\[ U = \]
The Complement of a Set

- The complement of set $A$, denoted $A^C$, is the set of all elements in a universal set, $U$, that are NOT in the set $A$. 
Set Intersection

- The *intersection* of sets $A$ and $B$, denoted $A \cap B$, is the set of all elements that belong to both set $A$ and to set $B$.
- The set of all elements that the original sets have in common.
Disjoint Sets

- If sets $A$ and $B$ have no elements in common, then $A \cap B = \emptyset$ and $A$ and $B$ are said to be disjoint sets.
Set Union

- The *union* of sets $A$ and $B$, denoted $A \cup B$, is the set of all elements that belong to either set $A$ or to set $B$ or to the both of them.
Example: What sets are equal to the following sets?

a) $U^c$

b) $\emptyset^c$

c) $(A^c)^c$

d) $A \cup A^c$

e) $A \cap A^c$
DeMorgan’s Laws

\[(A \cup B)^C = A^C \cap B^C\]
DeMorgan’s Laws

\[(A \cap B)^c = A^c \cup B^c\]
Example: Given the sets $U=\{1,2,\ldots,7\}$, $A=\{1,2,5,7\}$, $B=\{3,4,5\}$, and $C=\{1,7\}$, find the following sets:

a) $B^C$

b) $(A \cup C) \cap B$

c) $C^C \cap (A \cup B)^C$
Example: Let $U = \{x \mid x$ is a currently enrolled student at Idea University$\}$

$M = \{x \in U \mid x$ is a male$\}$

$B = \{x \in U \mid x$ is majoring in business$\}$

$C = \{x \in U \mid x$ is a member of the ROTC$\}$

Shade an appropriate Venn diagram to illustrate the following sets and then describe the sets in words.

a) $(M \cup C)^C$
Example continued:

b) \((M^C \cap C) \cup B\)
Example: Shade an appropriate Venn diagram illustrating the following sets:

a) \( A \cap B \cap C \)

b) \( A^c \cap B \cap C^c \)
Example continued:

c) \((A \cup B^C) \cap C^C\)