A function, $f$, is a rule that assigns to each value of $x$ one and only one value of $y$.

- $y = f(x)$
- $x$ is the independent variable, the “input”
- $y$ is the dependent variable, the “output”
- **Domain**: set of all $x$-values that make sense for the function

- **Range**: set of all $y$-values that result from the $x$-values in the domain
Linear Functions

- Of the form $f(x) = mx + b$
- $m$ and $b$ are constants.
- Use a linear function to create a linear model.
Mathematical Models

- Take real-world problems and solve them using mathematics.
- Be sure to define variables with appropriate units for all mathematical models.
- In the next few examples, we will use linear functions to create mathematical models.
Depreciation

- An item has an initial value and a final value.
- Assume the value decreases linearly with time.
- What does “linearly” mean?
  - General form: $V(t) = mt + b$
Example: Some computer equipment is originally purchased for $15,000. After 5 years, the equipment is worth $5000. Find an equation for the value, \( V \), of the equipment as a function of time, \( t \), in years.
What is the rate of depreciation of the equipment?

What is the equipment worth after 4 years?

Graph the depreciation equation.
Total Cost Function

- \( C(x) = cx + F \) gives the total cost of producing \( x \) items.
- Slope \( c \) represents the cost of making one additional item.
- Variable costs, \( cx \), depend on the number of items produced.
  - Examples include:

- \( F \) represents the fixed costs that are independent of the number of items produced.
  - Examples include:
Example: A company makes heaters. They find that the total cost to make 10 heaters is $1500 and the total cost to make 20 heaters is $1900. Find the total cost function. Identify the fixed and variable components.
Revenue Function

- $R(x) = sx$ gives the revenue (all the money brought in) from the sale of $x$ items.
- $s$ is the price per unit
Example: Find the revenue function for selling heaters if the heaters sell for $50 each. What is the revenue if 200 heaters are sold?
Profit Function

- $P(x) = R(x) - C(x)$ gives the profit from producing and selling $x$ items.
  - If revenue is greater than cost:
  - If cost is greater than revenue:
  - If cost and revenue are equal:
Example: Find the profit function for the production and sale of the heaters. What is the profit for the production and sale of 100 heaters?
Demand Function

- \( D(x) = p = mx + b \)
- Models the relationship between the price \( p \) per item and the number of items \( x \) purchased (demanded) by consumers.
Example: A store finds that it can sell 10 snowboards when the price is $450 each and 25 snowboards when the price is $300 each. Find the linear demand equation.
Supply Function

- $S(x) = p = mx + b$
- Models the relationship between the price $p$ per item and the number of items $x$ supplied to the market.
Example: A company manufactures snowboards. The company is not willing to sell snowboards if the unit price is $250 or lower. At $350 each, the company will supply 10 snowboards. Find the linear supply equation.
Equilibrium Point

- Represented by the point of intersection on the graph of the supply and demand equations.
- Shows the price and number of items that satisfy both the producer and consumer.
- The intersection point is \((x_0, p_0)\).
  - \(x_0\) is the equilibrium quantity
  - \(p_0\) is the equilibrium price
Example: What is the equilibrium point for the sale of snowboards?
Break-Even Analysis

- Point where a company makes no money, but also loses no money.
- Profit equals 0.
- Cost equals revenue.
Example: What is the break-even point for the company making heaters?
The Method of Least Squares

- Previously, we found a line to model a set of data using only two of the data points.
- If all the data points were used to find the equation of the line, it makes sense that the new line would be a better model.
- The Least Squares Method, via the graphing calculator, will do just this.
- Use the least-squares line, a.k.a. the regression line, for estimates.
**Example:** If the temperature is 75°F, how many cricket chirps per second would you expect to hear?

<table>
<thead>
<tr>
<th>Temperature (°F)</th>
<th>76</th>
<th>72</th>
<th>93</th>
<th>84</th>
<th>81</th>
</tr>
</thead>
<tbody>
<tr>
<td>Chirps per second</td>
<td>14</td>
<td>16</td>
<td>20</td>
<td>18</td>
<td>17</td>
</tr>
</tbody>
</table>
What is that ‘r’?

- Correlation coefficient
- Measures how close the data points are to the line.
- The closer the value is to $\pm 1$, the better the line fits the data.
- If the value is near 0, the data is not really linear.
Example: If you count 19 chirps per second, what must the temperature be?