Section F.3: Annuities and Sinking Funds

Section F.4: Present Value of Annuities and Amortization

Definition: An annuity is an instrument that involves fixed payments be made/received at equal time periods. An ordinary annuity is an annuity in which the payments are made at the end of each period. A simple annuity is an annuity where the payments coincides with the interest conversion period. In this class we work with simple ordinary annuities.

Formulas for simple ordinary annuities.

<table>
<thead>
<tr>
<th>Future Value</th>
<th>Sinking Fund</th>
</tr>
</thead>
<tbody>
<tr>
<td>$FV = PMT \frac{(1 + i)^n - 1}{i}$</td>
<td>$PMT = \frac{FV \times i}{(1 + i)^n - 1}$</td>
</tr>
</tbody>
</table>

account grows

<table>
<thead>
<tr>
<th>Present Value</th>
<th>Amortization</th>
</tr>
</thead>
<tbody>
<tr>
<td>$PV = PMT \frac{1 - (1 + i)^{-n}}{i}$</td>
<td>$PMT = \frac{PV \times i}{[1 - (1 + i)^{-n}]}$</td>
</tr>
</tbody>
</table>
1. Bob wants to deposit $100 at the end of every six months into an IRA that pays 7% compounded semiannually.

   A) How much money is in the account at the end of 25 years? \(13099.79\)

   B) How much money did Bob deposit into the account?

   C) How much interest did Bob get on the account?

\[
\begin{align*}
N &= 2(25) \\
I\% &= 7 \\
PV &= 0 \\
PMT &= -100 \\
FV &= ???
\end{align*}
\]

\[
\begin{align*}
&\text{B) } 100 \times 2 \times 25 = 5000 \\
&\text{C) } 13099.79 - 5000 = 8099.79
\end{align*}
\]

Oct 27-3:32 PM
2. Sue is starting her bank account with $1000 and will deposit $100 each month for the next 6 years. If the account pays 5.25% interest compounded monthly,

A) How much will she have in the account at the end of 6 years?
B) How much interest did she earn?
C) How much interest was earned in the third year?

\[ \begin{array}{c|c|c|c|c}
N & \text{12 (6)} & 36 & 24 & \text{put in } 1000 + 100(12)(6) \\
I\% & 5.25 & 5.25 & 5.25 & = 8200 \\
PV & -1000 & -1000 & -1000 & \text{Interest: } 9810.88 - 8200 \\
PMT & -100 & -100 & -100 & = 1610.88 \\
FV & ?? & ?? & ?? & \\
P/Y=C/Y & 12 & 12 & 12 & \\
\end{array} \]

1st yr. 2nd yr. 3rd yr. 4th yr.

\[ \begin{align*}
& N = 12(6) = 72 \\
& N = 12(3) = 36 \\
\end{align*} \]

Balance \( n = 36 \) - Balance \( n = 24 \) = change in the account in yr 3.

\[ \begin{align*}
5059.98 - 3635.17 & = 1424.81 \\
\end{align*} \]

\[ \begin{align*}
\text{Interest} & = \frac{\text{balance}}{n = 36} - \frac{\text{balance}}{n = 24} - \text{any payment} \\
& = \frac{5059.98 - 3635.17 - 100(12)}{12} \\
& = \$224.81 \\
\end{align*} \]
3. You have decided to set up a college fund for your kid. You have decided that $75,000 should be enough to get Jr most of the way through college. You decide to open the account when Jr is born and make monthly deposits every month for 18 years. The account pays interest at a rate of 4.75% compounded monthly.

A) What is your monthly deposit that would reach your goal?
B) What is the balance of the account after 10 years?
C) How much money did you deposit into the account?
D) How much interest did the account earn?

\[
\begin{array}{c|c|c}
N= & 12(18) & 10(12) \\
\%= & 4.75 & 4.75 \\
PV= & 0 & 0 \\
PMT= & ??? & -220.33 \\
FV= & 75000 & \?? \\
P/Y=C/Y= & 12 & 12 \\
\end{array}
\]

\[
\begin{align*}
&\text{C} = 220.33(12)(18) \\
&= $33,759.59 \\
&\text{PV} = -47,591.28 \\
&\text{FV} = 75000 - 47,591.28 \\
&= $27,408.72
\end{align*}
\]
4. You have decided that you want a new Chevy Truck in 5 years. It will cost $30,000. You are going to save up for the truck by making monthly payments into an account, paying 4.5% interest compounded monthly.

A) What monthly payments should you make so that you can buy the truck with cash? $446.79

B) What is the balance of the account after 25 payments? $11,687.14

\[ N = m \cdot t \]

\[ N = 12 \cdot 5 = 60 \]

\[ I\% = 4.5 \]

\[ PV = 0 \]

<table>
<thead>
<tr>
<th>PMT</th>
<th>[ 12 \left( \frac{25}{12} \right) = 25 ]</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>4.5</td>
</tr>
<tr>
<td></td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>-446.77</td>
</tr>
<tr>
<td>FV</td>
<td>30000</td>
</tr>
</tbody>
</table>

P/Y=C/Y = 12
5. You have decided that you want a new Chevy Truck. However you want it right now. It will cost $30,000.

A) What monthly payments should you make, paying 4.5% interest compounded monthly, so that you can payoff the truck in 5 years?

B) What is the balance of the account after 25 payments?

C) What payment would you have if there was a $9000 balloon payment at the end of the loan?
6. After you retire, you would like to receive quarterly payments of $5,000 for the next 15 years. The account earns 8% compounded quarterly.

A) What amount do you use to start the account? $173,804.43

B) How much interest was earned during the lifetime of the account?

Pull out $5,000 (4)(15) = 300,000
Put in $173,804.43

Interest = 300,000 - 173,804.43 = $126,195.57
7. You just won $4 million in the lottery, but you choose the payment method instead of the cash option. This means that you will receive 20 payments of $200,000. How much should be deposited into an account paying 9% compounded annually, so that you can receive the your payments for the next 19 years?

\[ \begin{align*}
N &= 19 \\
I\% &= 9 \\
PV &= ?? \\
PMT &= 200,000 \\
FV &= 0 \\
P/Y=C/Y &= 1
\end{align*} \]

$1,799,022.76
8. You want to buy a house. The house will cost $120,000. You put 5% down and then plan to pay off the house for the next 30 years with monthly payments. What payments should you make to pay off the loan if the interest rate is 7.5% compounded monthly? How much interest did you pay?

9. Repeat the house loan for a 15 year loan.
The rate per period is given by:
\[ r = \frac{R}{n} = \frac{0.75}{12} = 0.00625 \]

10. Write an amortization schedule for the first 4 months for problem 8.

<table>
<thead>
<tr>
<th>period</th>
<th>interest owed</th>
<th>payment</th>
<th>amt. toward principal</th>
<th>outstanding principal</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>114000</td>
</tr>
<tr>
<td>1</td>
<td>712.50</td>
<td>797.10</td>
<td>84.6</td>
<td>113915.4</td>
</tr>
<tr>
<td>2</td>
<td>711.97</td>
<td>85.13</td>
<td></td>
<td>113830.27</td>
</tr>
<tr>
<td>3</td>
<td>711.44</td>
<td>85.66</td>
<td></td>
<td>113744.61</td>
</tr>
<tr>
<td>4</td>
<td>A</td>
<td>B</td>
<td>C</td>
<td>D</td>
</tr>
</tbody>
</table>

- \[ A = d \times r \]
- \[ C = B - A \]
- \[ d = d - c \]
11. How much equity would you have on the 30 year note after 5 years?
   after 10 years?
   \[ \text{Equity} = \text{value of the object} - \text{what you still owe} \]
12. You had made payments on the 30 year mortgage for 7 years and then decide to refinance the loan for 15 years at 5.35%.

A) What would be the new house payment? $847.10

B) How much money would you save, not counting refinancing costs, by refinancing the loan?

\[
\begin{align*}
\text{old loan} & \quad \text{new loan} \\
N &= 7(12) & 15(12) \\
\% &= 7.5 & 5.35 \\
PV &= 114,000 & 810,469.130 \\
PMT &= -797.10 & ??? \\
FV &= ??? & 0 \\
P/Y = C/Y &= 12 & 12 \\
\text{still owe} &= 810,469.130
\end{align*}
\]

\[
\begin{align*}
\text{Paid on new loan} & \quad \text{Paid on old loan (remaining)} \\
847.10(12)(15) & \quad 797.10(12)(23) \\
= 6152478 & \quad = 219999.6 \\
\end{align*}
\]

\[
\begin{align*}
\text{Save} & \quad 219999.6 - 152478 = \underline{67521.60}
\end{align*}
\]
13. Your credit card has a balance of $2000 and the interest rate is 19.5% compounded monthly. If you only make the minimum monthly payment of $35 per month, how many years will it take to pay off the balance? How much interest did you pay? Assume that you are not making any more charges on the card.

\[ \text{N= ??} \rightarrow N = 163.719 \rightarrow N = 164 \text{ payments} \]

\[ > 13 \frac{3}{3} \text{ yrs.} \]
14. Mark has started to think about his retirement. He has decided that his goal is to receive monthly payments of $4,000 for 15 years when he retires at age 65. To achieve this goal, Mark is going to make monthly payments into his retirement account for forty years. He just turned 25. His retirement account earns interest at a rate of 6% per year compounded monthly. What payments should he make so that he will meet his goal? How much interest did Mark earn during the lifetime of the account?

\[ \begin{align*}
N &= 15(12) \\
I\% &= 6 \\
PV &= ??? \\
PMT &= 4000 \\
FV &= 479014.06 \\
P/Y = C/Y &= 12 \\
\end{align*} \]

He needs $479,014.06 to get payments of $4000 per month for 15 years. So, he put in $238.02 per month.

\[ \begin{align*}
pull \ out &= 4000(12)(15) = 720000 \\
pull \ in &= 238.02(12)(40) = 114249.44 \\
\text{Interest} &= 720000 - 114249.44 \\
&= 605750.56
\end{align*} \]