Section F.1: Simple Interest and Discounts

**Definition:** If the principal, $P$, is invested for a time period of $t$ at a simple interest rate of $r\%$ (for that period) then the interest earned at the end of the time period is given by

$$I = Prt$$

The future value, $A$ or $F$ of the investment at the end of the period is

$$A = P + I = P(1 + rt)$$

Example: You invest $500 at an annual simple interest rate of 4% for 6 years. How much interest did you earn? What is the balance at the end?

$$I = Prt = 500(0.04)(6) = 120$$

$$A = 500 + 120 = 620$$
Example: You invest $1000 at a monthly simple interest rate of 6.5% for 2 years. How much interest did you earn? What is the balance at the end?

\[
I = Prt = 1000 (0.065)(24) = 1560
\]

\[
A = 2560
\]
Example: You invest $2000 for 8 months and at the end of this time period you have earned $400 of interest. What is the annual simple interest rate? monthly simple interest rate?

\[ I = Prt \]

\[
\text{Annual} \quad \frac{400}{2000} = 0.20 r \left( \frac{r}{12} \right)
\]

\[ r = 0.25 \]

30% 

\[ A = P(1+rt) \]

\[
\text{Monthly} \quad \frac{400}{2000} = 0.20 r \left( \frac{8}{12} \right)
\]

\[ r = 0.025 \]

2.5%
Example: You want to have a total of $3000 at the end of 3 years. If the account has an annual simple interest rate of 4.5%, what amount do you have to invest to meet your goal?

\[ A = P(1 + rt) \]

\[ 3000 = P (1 + 0.045(3)) \]

\[ P = \$2643.17 \]
**Definition:** The discount, $D$, on a discounted loan of $M$ dollars at a simple interest rate of $r\%$ for the time period $t$ is

$$D = Mrt$$

The **Proceeds**, $P$, of the loan is the actual amount the borrower receives from the loan is given by

$$P = M - D = M(1 - rt)$$

**Example:** John will pay back a $10,000 loan at the end of 15 months. This loan has an annual simple discount rate of 7%. What is the discount on the loan and How much money does John actually receive from the bank?

\[
D = Mrt \\
= 10000 \times (0.07) \times \left( \frac{15}{12} \right) \\
= 875
\]

\[
P = M - D = 10000 - 875 = 9,125
\]
Example: Susan needs $1,200 right now. She has agreed to take out a
discount loan from bank and repay it is 2 years. If the annual discount
rate is 8%, compute the maturity value of the loan.

\[ D = m \cdot t \quad P = m \left( 1 - r \cdot t \right) \]

\[ 1200 = m \left( 1 - (0.08) (2) \right) \]

\[ m = \$1428.57 \]

Example: What is the effective rate of interest for the loan in the pre-
vious example?

\[ \text{If the account earned simple interest for the time} \]

\[ \text{period what would the rate be.} \]

\[ r_{\text{eff}} = \frac{r}{1 - rt} = \frac{0.08}{1 - 0.08(2)} = 0.09524 \]

9.524% eff yield.

annual simple interest rate for 2 yrs.
**Definition:** The effective interest rate, effective yield, on a discount loan of length $t$ years with an annual discount rate is given by

$$r_{eff} = \frac{D}{Pt} = \frac{m \cdot r \cdot t}{m(1-r\cdot t) \cdot t} = \frac{r}{1-r\cdot t}$$

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**Example:** What is effective yield on a discount loan with an annual simple interest rate of 4% when the loan is due in 4 months?

$$r_{eff} = \frac{.04}{1-.04\left(\frac{4}{12}\right)} = 4.054\%$$