Section 3.4: The Normal Distribution

To find probability with a continuous random variable $X$, we use a probability density function, $f(x)$. This function has the properties: 1) $f(x) \geq 0$ for all values of $X$ and 2) the area under the graph of $f(x)$ is equal to 1 (on the values of $X$).

Normal Curve:

This distribution is always centered around the mean. The standard deviation regulates how high of a peak the curve will have. The formula for this curve is

$$f(x) = \frac{1}{\sigma \sqrt{2\pi}} e^{\left(\frac{-(x-\mu)^2}{2\sigma^2}\right)}$$
Shade the area under the normal curve that represents these probabilities.

\[ P(X > a) \]

\[ P(c < X < d) \]

\[ P(X = a) = 0 \]
**Definition:** The standard normal curve is the normal curve with \( \mu = 0 \) and \( \sigma = 1 \). The random variable for the standard normal curve is \( Z \).

Note: This is the curve that is used to create the normal distribution charts found in the back of the math book. These charts will not be used in this course. To convert to the standard normal curve use the formula \( Z = \frac{z-\mu}{\sigma} \).

\[
\begin{align*}
\Phi(z \leq 0.27) &= 0.6064 \\
\Phi(z > 0.43) &= 1 - 0.6664
\end{align*}
\]

<table>
<thead>
<tr>
<th>( z )</th>
<th>.00</th>
<th>.01</th>
<th>.02</th>
<th>.03</th>
<th>.04</th>
<th>.05</th>
<th>.06</th>
<th>.07</th>
<th>.08</th>
<th>.09</th>
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</thead>
<tbody>
<tr>
<td>0.0</td>
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<td>5080</td>
<td>5120</td>
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<td>5199</td>
<td>5239</td>
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<td>5478</td>
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<td>5557</td>
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<td>5636</td>
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<td>5714</td>
<td>5753</td>
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<td>6064</td>
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<td>6628</td>
<td>6664</td>
<td>6700</td>
<td>6736</td>
<td>6772</td>
<td>6808</td>
<td>6844</td>
<td>6879</td>
</tr>
</tbody>
</table>
Calculator commands

The TI-84 has built in commands that we will use to work with normal distributions. They can be found in the Distribution menu by pressing \(2\text{nd} \text{ VARS}\). **Note:** to enter \(1E99\) press \(1 \text{ EE } 9 \text{ EE } 9\). The \(E\) represents scientific notation and the \(\text{ EE} \) is found by pressing \(2\text{nd} \), .

\text{normalcdf(lower, upper, } \mu, \sigma) \text{ computes the probability that a continuous R.V. } X \text{ is between the lower bound and the upper bound.}

<table>
<thead>
<tr>
<th>Calculate</th>
<th>Lower</th>
<th>Upper</th>
</tr>
</thead>
<tbody>
<tr>
<td>(P(X &lt; a))</td>
<td>(-1E99)</td>
<td>(a)</td>
</tr>
<tr>
<td>(P(X &gt; a))</td>
<td>(a)</td>
<td>(1E99)</td>
</tr>
<tr>
<td>(P(a &lt; X &lt; b))</td>
<td>(a)</td>
<td>(b)</td>
</tr>
</tbody>
</table>

\text{invnorm(area, } \mu, \sigma) \text{ will return a value } A \text{ that satisfies the equation } P(X < A) = \text{ area.}
Example: Let $X$ be a continuous random variable that is normally distributed with $\mu = 53$ and $\sigma = 4$. Compute the following.

A) $P(45 < X < 58)$

B) $P(X < 56)$

C) $P(X > 38)$

Example: Compute $P(0.5 < Z < 2)$
Example: The length of what are considered "one-inch" bolts is found to be a normally distributed random variable with a mean of 1.001 inches and a standard deviation 0.002 inches. If a bolt measures more than two standard deviations from the mean, it is rejected as not meeting factory tolerances. What percentage of the bolts will the factory reject?

\[
\mu = 1.001 \\
\sigma = 0.002
\]

\[
1 - \text{ncdf} (0.999, 1.001, 0.002) = 0.0455
\]

0.55%
Example: A particular brand of dishwasher has a life expectancy that is estimated to be normally distributed, with a mean of 10 years, 8 months and a standard deviation of 1 year, 2 months. Suppose that such dishwashers are guaranteed to last 9 years. Of every 250 sold, how many will fail to last through the guarantee period?

\[
\mu = 128 \text{ mo.} \\
\sigma = 14 \text{ mo.}
\]

\[
P(X < 108) = \text{ncdf}(-15.79, 108, 128, 14) = 0.0766
\]

\[0.0766 \times 250 = 19.15 \approx 19.
\]
Example: The weight of infants is normally distributed with a mean of 7.4 lbs and a standard deviation of 1.2 lbs. Fifty infants are selected at random, find the probability that at most 10 of them weigh between 7 and 8 pounds.

\[ \mu = 7.4 \]
\[ \sigma = 1.2 \]

\[ \text{n} = 50 \]
\[ r = 0, 1, 2, \ldots, 10 \text{ success} \]

\[ P = .322 \]

\[ \text{bcdf}(50, .322, 10) = .0410 \]
Example: Let $X$ be a normally distributed random variable with $\mu = 53$ and $\sigma = 4$.

A) Find $A$ such that $P(X < A) = 0.356$

B) Find $B$ such that $P(X > B) = 0.525$
Example: Find J such that $P(-J < Z < J) = 0.78$

$J = \text{inv norm (.11 + .78, 0, 1)}$

$= 1.2265$
Example: A prof is going to grade an exam on a bell curve. The prof decided that the top 10% of the bell curve will be an A, the next 15% a B, the next 30% a C, and the next 20% a D. If the mean for the class is 70 and the standard deviation is 18, find the lowest grade on the exam that will get you an A and the lowest grade on the exam that will get you a B.

\[ A = \text{invnorm}(0.9, 70, 18) = 93.0679 \rightarrow 94 \]

\[ B = \text{invnorm}(0.75, 70, 18) = 82.1408 \rightarrow 83 \]