Section 3.1: Random Variables and Histograms

Definition: A random variable is a rule that assigns a real number to each outcome of a sample space.

Example: Let $X$ be the number of boys in a 3 kid family.

$S = \{ \text{bbb, bbg, bgb, gbb, ggb, gbg, bgg, ggg} \}$

A) What are the values of the random variable $X$?

B) Give the probability distribution for $X$.

<table>
<thead>
<tr>
<th>$X$</th>
<th>$0$</th>
<th>$1$</th>
<th>$2$</th>
<th>$3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>prob</td>
<td>$\frac{1}{8}$</td>
<td>$\frac{3}{8}$</td>
<td>$\frac{3}{8}$</td>
<td>$\frac{1}{8}$</td>
</tr>
</tbody>
</table>
Types of Random Variables

discrete: skips values.
finite: fixed # of values.
infinite: infinite # of values.

continuous $\Rightarrow$ doesn't skip values.
time, distance, weight, ...
Example: Classify these random variables. Give the values of the random variable.

A) $X =$ the number of hours you sleep in a day.

\[0 \leq X \leq 24\]

B) $X =$ the number of good jokes/puns that I tell in a semester.

\[
\text{discrete } \begin{cases} \text{finite} \end{cases} \quad x = 0, 1, 2, \ldots, 5
\]

C) $X =$ the number of rolls it takes to get a 5 on a 10-sided die.

\[
\text{discrete } \begin{cases} \text{infinite} \end{cases} \quad x = 1, 2, \ldots
\]
D) $X =$ the number of draws it takes to get an Ace when drawing cards from a standard deck of cards without replacement.

\[ X = 1, 2, \ldots, 49 \]

E) $X =$ the number of yellow balls drawn in a sample of 6 from a box that contains 5 yellow balls, 2 green balls, 1 red ball and 1 purple ball.

\[ X = x \text{ with } x = 0, 1, 2, 3, 4, 5, 6 \]
Example: Let $X$ = the number of clubs in a five card hand.

Find $P(X = 2) = \frac{\binom{13}{2} \cdot \binom{39}{3}}{\binom{52}{5}}$
**Definition:** A **histogram** is a way to present the probability distribution of a discrete random variable.

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**Example:** Draw the probability distribution $X$.  

<table>
<thead>
<tr>
<th>$X$</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>prob.</td>
<td>0.2</td>
<td>0.1</td>
<td>0.15</td>
<td>0.4</td>
<td>0.15</td>
</tr>
</tbody>
</table>
Example: The following histogram is only missing the rectangle at $x = 7$.

A) Find $P(X = 7)$

B) Give the probability distribution for $X$.

C) Find $P(2 \leq X < 6) =$
Example: A task consist of drawing a ball from a box containing 2 red, 3 green and 1 white and replacing it after the color is noted.. Let the random variable $X$ be the number of green balls drawn when this task is repeated 6 times.

A) Compute $P(X = 2)$

B) Compute $P(X < 5)$

**Definition:** Given a sequence of $n$ Bernoulli trials with the probability of success $p$ and the probability of failure of $q$, the binomial distribution, for $k = 0, \ldots, n$, is given by

$$P(X = k) = C(n, k)p^k q^{n-k}$$
Example: A cookie company wants to check the consistency of the number of raisins in its oatmeal raisin cookies. A few cookies from each batch are selected and the number of raisins are counted. After several days, the following results were found.

<table>
<thead>
<tr>
<th>number of cookies</th>
<th>3</th>
<th>5</th>
<th>6</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>number of raisins</td>
<td>2</td>
<td>7</td>
<td>9</td>
<td>10</td>
</tr>
</tbody>
</table>

Define the random variable and give the probability distribution.