Section 1.4: Basic Probability and Section 1.5: Rules for Probability

Standard Deck of Cards
A deck of cards has 4 suits: diamonds, hearts, clubs, and spades. The suits of diamonds and hearts are both red and the suits of clubs and spades are both black. Each suit has the following denominations: Ace, 2, 3, 4, 5, 6, 7, 8, 9, 10, Jack, Queen, and King.

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face cards.
Definition: Probability is a number that is assigned to an outcome of a sample space that indicates how likely that outcome is to happen when conducting an experiment.

Rules of Probability

1) all prob is between 0 + 1 inclusive.

2) all prob adds up to 1

<table>
<thead>
<tr>
<th>Die Roll</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Method A</td>
<td>1/6</td>
<td>1/6</td>
<td>1/6</td>
<td>1/6</td>
<td>1/6</td>
<td>1/6</td>
</tr>
<tr>
<td>Method B</td>
<td>0.1</td>
<td>0.2</td>
<td>0.3</td>
<td>0.2</td>
<td>0.1</td>
<td>0.3</td>
</tr>
<tr>
<td>Method C</td>
<td>0.1</td>
<td>0.2</td>
<td>0.3</td>
<td>0.2</td>
<td>0.1</td>
<td>0.1</td>
</tr>
</tbody>
</table>

fair die

more than 1.

rigged die
Definition: A **probability distribution** is a chart that shows the probability of every outcome in the sample space.

Definition: A sample space where every outcome has the same probability (chance of happening) is called a **uniform sample space** and

\[ P(\text{any individual outcome}) = \frac{1}{n(S)} \]
Example: A group of people were asked what was their favorite soft drink. The results of the survey are given in the table.

<table>
<thead>
<tr>
<th>drinks</th>
<th>Dr Pepper</th>
<th>Coke</th>
<th>Pepsi</th>
<th>Rootbeer</th>
<th>other</th>
</tr>
</thead>
<tbody>
<tr>
<td>people</td>
<td>175</td>
<td>10</td>
<td>40</td>
<td>60</td>
<td>25</td>
</tr>
</tbody>
</table>

Give the probability distribution that is associated with this survey.

Example: The number of grades for a group of Math 166 students are shown in the table.

<table>
<thead>
<tr>
<th>grade</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>F</th>
</tr>
</thead>
<tbody>
<tr>
<td>frequency</td>
<td>40</td>
<td>84</td>
<td>71</td>
<td>38</td>
<td>31</td>
</tr>
</tbody>
</table>

\[ \text{Total 264} \]

If a student from this group is selected at random, what is the probability that the student made

A) an A?

\[ \frac{40}{264} \]

B) a B?

\[ \frac{84}{264} \]
Example: You flip a coin 10, 50, and then 100 times. For each of these, how many times will you get a head?

Definition: If E is an event of a sample space, then

\[ P(E) = \sum \text{prob of each outcome in } E. \]

\[ P(S) = 1 \]

\[ P(\emptyset) = 0 \]

If S is uniform

\[ P(E) = \frac{n(E)}{n(S)} \]
Example: A single card is drawn from a standard deck of cards. Find these probabilities.

A) \( P(\text{king}) = \frac{4}{52} \)

B) \( P(\text{heart}) = \frac{13}{52} \)

C) \( P(\text{heart or a king}) = \frac{13 + 4 - 1}{52} \)

D) \( P(\text{not a king or not a queen}) = \frac{48}{52} + \frac{48}{52} - \frac{4}{52} = 1 \)

\[ P(A \cup B) = P(A) + P(B) - P(A \cap B) \]
Example: Roll two six sided die (one red and one green). Give the probability distribution for the sum of the die.

<table>
<thead>
<tr>
<th></th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>11</th>
<th>12</th>
</tr>
</thead>
<tbody>
<tr>
<td>red</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>1/36</td>
<td></td>
<td>2/36</td>
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<td></td>
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<td>3/36</td>
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<td></td>
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<td></td>
<td>1/36</td>
<td></td>
<td>4/36</td>
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<td></td>
<td>2/36</td>
<td></td>
<td>1/36</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>1</th>
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<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>11</th>
<th>12</th>
</tr>
</thead>
<tbody>
<tr>
<td>green</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>1/36</td>
<td></td>
<td>2/36</td>
</tr>
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<td>2/36</td>
<td></td>
<td>3/36</td>
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<td></td>
<td></td>
<td>1/36</td>
<td></td>
<td>4/36</td>
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<td></td>
<td></td>
<td></td>
<td></td>
<td>2/36</td>
<td></td>
<td>1/36</td>
</tr>
</tbody>
</table>

Example: Roll a 6 sided die. Suppose that any even number is twice as likely to happen as any odd number. Find the probability distribution.

\[
\begin{array}{cccccc}
  & 1 & 2 & 3 & 4 & 5 & 6 \\
 P & \frac{1}{9} & \frac{2}{9} & \frac{1}{9} & \frac{2}{9} & \frac{1}{9} & \frac{2}{9} \\
\end{array}
\]

\[p(2) = 2, p(1) = 2, p(3) = 2, p(5)\]

\[p(2) = p(1) = p(3) = p(5) = x\]

\[p(2) + p(2) + p(3) + p(4) + p(5) + p(6) = 1\]

\[x + 2x + x + 2x + x + 2x = 1\]

\[9x = 1\]

\[x = \frac{1}{9}\]
Example: A four sided die is rolled. If a four or a one is rolled the die is rolled a second time. The total sum of the numbers rolled is recorded. Give the probability distribution.
Example: Use the probability distribution and the events to answer these questions.

<table>
<thead>
<tr>
<th></th>
<th>a</th>
<th>b</th>
<th>c</th>
<th>d</th>
<th>e</th>
<th>f</th>
</tr>
</thead>
<tbody>
<tr>
<td>S</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>prob</td>
<td>0.15</td>
<td>0.08</td>
<td>0.21</td>
<td>0.12</td>
<td>0.25</td>
<td>.19</td>
</tr>
</tbody>
</table>

\[ E = \{ a, c, d, e\} \quad F = \{b, d, f\} \quad G = \{a, b, d\} \]

A) \( P(S) = 1 \quad P(\varnothing) = 0 \quad P(\{f\}) = .19 \)

B) \( P(E) = .15 + .21 + .12 + .25 = .73 \)

C) \( P(E^c) = .08 + .19 = .27 \)

\[ P(E) + P(E^c) = 1 \]
\[
\begin{array}{c|cccccc}
S & S_1 & S_2 & \varepsilon & S_3 & \varepsilon & \varepsilon \\
\hline
\text{prob} & 0.15 & 0.08 & 0.21 & 0.12 & 0.25 & 0.19 \\
\end{array}
\]

\[
E = \{a, c, d, e\} \quad F = \{b, d, f\} \quad G = \{a, b, d\}
\]

D) \( P(F \cap G) = 0.08 + 0.12 = 0.2 \)

E) \( P(E \cup G) = P(E) + P(G) - P(E \cap G) \)

\[
= 0.73 + (0.15 + 0.08 + 0.12) - (0.15 + 0.12)
\]

\[
= 0.73 + 0.35 - 0.27 = 0.81
\]

\[
P(E \cup G) = 1 - 0.19 = 0.81
\]
Example: It is know from a survey that 29% of the people buy product A, 36% of the people buy product B and 11% buy both products. Find the probability that a person selected at random

A) buys only one of the product.

\[0.18 + 0.25\]

B) doesn’t buy either of the products.

\[0.46\]
Example: This table classifies the English, History, Math, and Poly Sci majors at State U according to their year. (There are no double majors.)

<table>
<thead>
<tr>
<th></th>
<th>Freshmen(F)</th>
<th>Sophomores(Soph)</th>
<th>Juniors(J)</th>
<th>Seniors(Sr)</th>
<th>Totals</th>
</tr>
</thead>
<tbody>
<tr>
<td>English(E)</td>
<td>64</td>
<td>35</td>
<td>31</td>
<td>41</td>
<td>171</td>
</tr>
<tr>
<td>History(H)</td>
<td>55</td>
<td>41</td>
<td>33</td>
<td>52</td>
<td>181</td>
</tr>
<tr>
<td>Math(M)</td>
<td>29</td>
<td>32</td>
<td>50</td>
<td>69</td>
<td>180</td>
</tr>
<tr>
<td>Poly Sci(PS)</td>
<td>70</td>
<td>33</td>
<td>41</td>
<td>37</td>
<td>181</td>
</tr>
<tr>
<td>Totals</td>
<td>218</td>
<td>141</td>
<td>155</td>
<td>199</td>
<td>713</td>
</tr>
</tbody>
</table>

If a student is selected at random, find the probability that

A) The student is a History major and a Sophomore. \( \frac{41}{713} \)

B) The student is not a Sophomore and is an English major.
\[
\frac{64 + 31 + 41}{713} = \frac{171 - 35}{713}
\]

C) The student is a Math major or is a Senior.
\[
\frac{29 + 32 + 50 + 69 + 41 + 52 + 37}{713} = \frac{199 + 180 - 69}{713}
\]
Example: Roll a 4 side die and a 6 sided die. Find the probability of

A) getting a sum greater than 9.

\[ \frac{1}{24} \]

B) getting a sum of 6 and at least one die comes up a 2.

\[ \frac{2}{24} \]

C) getting a sum of 6 or at least one die comes up a 2.

\[ \frac{11}{24} \]
**Definitions:** The odds that an event $E$ will occur, odds in favor of $E$, are given as $a$ to $b$ or $a:b$ where $a$ and $b$ are integers and the fraction $a/b$ is in reduced form.

Odds in favor of $E$ are 4 to 7

For every 4 times $E$ happens there are 7 times it doesn't.

$$P(E) = \frac{4}{7+4} = \frac{4}{11}$$
Example: The odds in favor of the event $E$ are 3 to 8.

A) Find the odds against $E$. $\frac{8}{3}$

B) Find $P(E)$ and $P(E^c)$.

\[ \begin{align*}
P(E) &= \frac{3}{3+8} = \frac{3}{11} \\
P(E^c) &= \frac{8}{3+8} = \frac{8}{11}
\end{align*} \]

Example: The odds in favor of Joe telling a funny joke (in class) is 12 to 5. Find the probability of Joe telling a funny joke.

\[ \frac{12}{12+5} \]
Computing odds from probability

\[ \text{find odds in favor of } E \]

**Step 1** \[ \frac{P(E)}{P(E^c)} \]

**Step 2** Reduce to \( \frac{a}{b} \)

**Step 3** answer is \( \frac{a}{b} \)

Example: If \( P(E) = 0.18 \), find the odds in favor of \( E \).

\[ \frac{0.18}{0.82} = \frac{9}{41} \]

answer: 9 to 41
Example: If \( P(F) = 0.32 \), find the odds against \( F \).

\[
\frac{0.32}{0.68} = \frac{8}{17} \quad \Rightarrow \quad \text{in favor is 8 to 17}
\]

\[\text{answer is 17 to 8}\]

Example: A card is drawn from a standard deck of cards. Find the odds that the card was a face card.

\[
\frac{12}{52} = \frac{12}{40} = \frac{6}{20} = \frac{3}{10}
\]

\[\text{3 to 10}\]

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