Section 1.2: The Number of Elements in a Set/Venn Diagrams

Example: If $A = \{1, 3, 5\}$ and $B = \{3, 4, 5, 6\}$ compute the following.

\[
\begin{align*}
n(A) &= \frac{3}{\text{3}} \\
n(B) &= \frac{4}{\text{4}} \\
n(A \cap B) &= \frac{2}{\text{2}} \\
n(A \cup B) &= 3 + 4 - 2 = 5
\end{align*}
\]

$A \cap B = \{3, 5\}$

Union Formulas:

\[
n(A \cup B) = n(A) + n(B) - n(A \cap B)
\]

If $A$ and $B$ are disjoint,

\[
n(A \cup B) = n(A) + n(B)
\]
Example: Suppose we survey 60 shoppers about two products: A and B.

30 shoppers bought product A
20 shoppers bought product B
9 shoppers bought product A and product B

A) Fill in a Venn Diagram that represents this information.
B) How many shoppers bought product A but not B?

$$21$$

C) How many shoppers bought product A or B?

$$21 + 9 + 11$$

D) How many shoppers bought at least one product?

$$21 + 9 + 11$$

E) How many shoppers bought exactly one product?

$$21 + 11$$

F) How many shoppers bought neither of these products?

$$19$$
Example: If \( n(U) = 500 \) and \( n(A \cup B) = 300 \), what is \( n(A^c \cap B^c) \)?

\[
500 - 300 = 200
\]
Example: Supposed we poll 100 people with 3 yes/no questions.

22 people answered no to all three questions.
45 answered yes to question 1.
25 answered yes to question 2.
32 answered yes to question 3.
6 answered yes to only question 1 and question 3.
12 answered yes to question 2 and question 3.
2 answered yes to all three questions.

A) Fill in a Venn Diagram that represents this information.
B) How many people answered yes to question 2 or question 3?

\[4 + 9 + 2 + 10 + 6 + 14\]

C) How many people answered yes to at most one question?

\[33 + 9 + 14 + 22\]

D) How many people answered yes to question 2 but not question 1?

\[9 + 10\]

E) How many people answered yes to exactly 2 questions?

\[6 + 4 + 10\]
De Morgan's Laws:

\[(A \land B)^c = A^c \lor B^c\]

\[(A \lor B)^c = A^c \land B^c\]