Section F.2: Compound Interest

Definition: Suppose the principal, $P$, is invested for $t$ years at an annual interest rate of $r\%$ and interest is compounded $m$ times per year. The future amount, $A$ or $F$, is given by

$$A = P(1 + i)^n = P \left(1 + \frac{r}{m}\right)^{mt}$$

$$i = \frac{r}{m} \quad \text{rate per period.}$$

$$n = mt \quad \text{total number of periods for the account.}$$
Example: Find the balance of the account if you invest $600 for 7 years at a nominal rate of 5% compounded

A) annually.
\[ A = 600 \left( 1 + \frac{.05}{1} \right)^{1 \cdot 7} = 844.26 \]

B) semiannually.
\[ A = 600 \left( 1 + \frac{.05}{2} \right)^{2 \cdot 7} = 847.78 \]

C) quarterly.
\[ A = 600 \left( 1 + \frac{.05}{4} \right)^{4 \cdot 7} = 849.60 \]

D) monthly.
\[ A = 600 \left( 1 + \frac{.05}{12} \right)^{12 \cdot 7} = 850.82 \]

E) daily.
\[ A = 600 \left( 1 + \frac{.05}{365} \right)^{365 \cdot 7} = 851.42 \]
Example: You want $2000 in an account at the end of 3 years. If the account gets a nominal rate of 5.75% compounded quarterly, how much do you start the account with?

\[ A = P \left( 1 + \frac{r}{n} \right)^{nt} \]

\[ 2000 = P \left( 1 + \frac{0.0575}{4} \right)^{4 \cdot 3} \]

\[ P = \frac{2000}{\left( 1 + \frac{0.0575}{4} \right)^{12}} = \$1685.18 \]
Example: You have the choice of investing money in one of two different accounts. The first account is at Bank A and has a rate of 6.51% compounded semiannually. The second account is at Bank B and has a rate of 6.08% compounded daily. Which account is the better deal?

\[ r_{\text{eff}}^A = 100 \left( 1 + \frac{0.0651}{2} \right)^2 - 100 = 6.61595\% \]

\[ r_{\text{eff}}^B = 6.26097\% \]

These are the rates if the account earns simple interest for 1 yr.

Definition: For compound interest, the effective yield, \( r_{\text{eff}} \), is given by

\[ r_{\text{eff}} = 100 \left( 1 + \frac{r}{m} \right)^m - 100 \]

\( r_{\text{eff}}(r\%, m) \) gives answer as \( \% \)
Example: You invest $2000 in an account that pays interest compounded monthly. What interest rate do you need to have a balance of $5,000 at the end of 3 years.

\[ A = P \left( 1 + \frac{r}{n} \right)^{nt} \]

\[ 5000 = 2000 \left( 1 + \frac{r}{12} \right)^{36} \]

\[ 2.5 = \left( 1 + \frac{r}{12} \right)^{36} \]

\[ (2.5)^{\frac{1}{36}} = 1 + \frac{r}{12} \]

\[ \frac{(2.5)^{\frac{1}{36}} - 1}{\frac{1}{36}} = \frac{r}{12} \]

\[ r = 12 \left( (2.5)^{\frac{1}{36}} - 1 \right) \]

\[ r = 30.935\% \]
Example: A zero coupon bond will mature in 5 years and has a face value of $8,000. If the bond has a return of 4.75% compounded annually, how much should you pay for it?

\[ n = 15 \]
\[ i = 4.75 \%
\[ P = \? \%
\[ Pmt = 0 \]
\[ FV = 8000 \]
\[ P/Y = C/Y = 1 \]
TVM Solver

The TVM solver that is built function on the TI-83/84 calculators. If you are using the old TI-83 press 2nd [x-1] and then press [ENTER], otherwise press the [APPS] and the select the Finance application and press enter. Here are the variables that are used in the TVM Solver.

\[ N = m \times t \] which is the total number of periods (compounding) for the life of the account.

\[ I\% = \] The interest rate per year as a percentage.

\[ PV = \] The present value (starting value) of the account.

\[ PMT = \] This is the payment that is made each period.

\[ FV = \] The future value (end value) of the account.

\[ P/Y = \] The number of payments per year.

\[ C/Y = \] The number of compoundings per year.

For this class, \( P/Y \) and \( C/Y \) are equal and \textbf{PMT:END BEGIN} should be set to \textbf{END}.