Section F.1: Simple Interest and Discounts

Definition: If the principal, \( P \), is invested for a time period of \( t \) at a simple interest rate of \( r\% \) (for that period) then the interest earned at the end of the time period is given by

\[ I = Prt \]

The future value, \( A \) or \( P \), of the investment at the end of the period is

\[ A = P + I = P(1 + rt) \]

Example: You invest $500 at an annual simple interest rate of 4% for 6 years. How much interest did you earn? What is the balance at the end?

\[ I = Prt \]
\[ I = 500 \times 0.04 \times 6 \]
\[ = 5 \times 120 \]
\[ = 620 \]

\[ A = P + I = 620 \]
Example: You invest $1000 at a monthly simple interest rate of 6.5% for 2 years. How much interest did you earn? What is the balance at the end?

\[ I = Prt \]
\[ = 1000 \times (0.065) \times (24) \]
\[ = 1560 \]

\[ A = 12560 \]
Example: You invest $2000 for 8 months and at the end of this time period you have earned $400 of interest. What is the annual simple interest rate? monthly simple interest rate?

\[
I = Prt
\]

Annual
\[400 = 2000r \left( \frac{8}{12} \right)\]

\[r = 0.3\]

Answer: 30%  

\[
A = P(1+rt)
\]

Monthly
\[400 = 2000r (8)\]

\[r = 0.025\]

Answer: 2.5%
Example: You want to have a total of $3000 at the end of 3 years. If the account has an annual simple interest rate of 4.5%, what amount do you have to invest to meet your goal?

\[ I = Prt \quad A = P(1 + rt) \]

\[ 3000 = P \left( 1 + \left(0.045\right)(3) \right) \]

\[ P = 2643.17 \quad \text{Present value at the $3000} \]
**Definition:** The discount, $D$, on a discounted loan of $M$ dollars at a simple interest rate of $r\%$ for the time period $t$ is

$$D = Mrt$$

The **Proceeds**, $P$, of the loan is the actual amount the borrower receives from the loan is given by

$$P = M - D = M(1 - rt)$$

Example: John will pay back a $10,000 loan at the end of 15 months. This loan has an annual **simple discount rate** of 7%. What is the discount on the loan and how much money does John actually receive from the bank?

$$M = 10000 \quad r = 7\% \quad t = \frac{15}{12}$$

$$D = Mrt$$

$$= 10000 \times 0.07 \times \left(\frac{15}{12}\right)$$

$$D = 875$$  \text{- Interest paid on the loan.}

$$P = M - D = 10000 - 875 = 9125$$  \text{- what John takes home}
\[ D = mrt \quad P = m(1 - rt) \]

Example: Susan needs $1,200 right now. She has agreed to take out a discount loan from bank and repay it in 2 years. If the annual discount rate is 8%, compute the maturity value of the loan.

\[ P = 1200 \quad r = 8\% \quad t = 2 \quad \text{Interest} = 228.57 \]

\[ P = m(1 - rt) \]

\[ 1200 = m \left( 1 - (0.08)(2) \right) \]

\[ m = 1428.57 \]

Example: What is the effective rate of interest for the loan in the previous example?

If this account earned simple interest for the same time period it would be \( \text{Reff} \).

From next page \( \text{Reff} = 9.52375\% \)
\[ I = 228.57 \]

Account start = 1200 = \( P \)

Account end = 1428.57 = \( A \)

\[ t = 2 \text{ yrs.} \]

\[ I = P r t \]

\[ 228.57 = 1200 \cdot r \cdot (2) \]

\[ \text{Simple interest rate} = 9.52375\% \]
Definition: The effective interest rate, effective yield, on a discount loan of length \( t \) years with an annual discount rate is given by

\[
\begin{align*}
    r_{eff} = \frac{D}{P_t} &= \frac{m \cdot r \cdot t}{P \cdot r \cdot t} = \frac{m \cdot r \cdot t}{m(1-r)t} \cdot \frac{1}{t} = \frac{r}{1-rt}
\end{align*}
\]

\[
\begin{align*}
    D = m \cdot r \cdot t \\
    P = m(1-rt)
\end{align*}
\]

Example: What is effective yield on a discount loan with an annual simple interest rate of 4% when the loan is due in 4 months?

\[
\begin{align*}
    r_{eff} = \frac{.04}{1-.04(\frac{4}{12})} = .04054 \rightarrow 4.054\% 
\end{align*}
\]