Section 3.1: Random Variables and Histograms

Definition: A random variable is a rule that assigns a real number to each outcome of a sample space.

Example: Let $X$ be the number of boys in a 3 kid family.

$S = \{ \text{bbb, bbg, bgb, gbb, gbg, bgg, bgg, ggg} \}$

A) What are the values of the random variable $X$?

B) Give the probability distribution for $X$. 

<table>
<thead>
<tr>
<th>$X$</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Prob.</td>
<td>$\frac{1}{8}$</td>
<td>$\frac{3}{8}$</td>
<td>$\frac{3}{8}$</td>
<td>$\frac{1}{8}$</td>
</tr>
</tbody>
</table>
Types of Random Variables

\begin{itemize}
  \item \textit{discrete}: skip values
  \item \textit{finite}:
  \item \textit{infinite}:
  \item \textit{continuous}: don't skip values.
\end{itemize}

\textit{time, distance, weight, ...}
Example: Classify these random variables. Give the values of the random variable.

A) $X =$ the number of hours you sleep in a day.

\[ 0 \leq x \leq 24 \]

B) $X =$ the number of good jokes/puns that I tell in a semester.

\[ \text{discrete finite} \quad x = 0, 1, 2, \ldots, 5 \]

C) $X =$ the number of rolls it takes to get a 5 on a 10-sided die.

\[ \text{discrete infinite} \quad x = 1, 2, 3, \ldots \]
D) $X =$ the number of draws it takes to get an Ace when drawing cards from a standard deck of cards without replacement.

$X \in \{1, 2, \ldots, 49\}$

E) $X =$ the number of yellow balls drawn in a sample of 6 from a box that contains 5 yellow balls, 2 green balls, 1 red ball and 1 purple ball.

$X = \{1, 2, 3, 4, 5, 6\}$
Example: Let $X = $ the number of clubs in a five card hand.

Find $P(X = 2) = \frac{\binom{13}{2} \cdot \binom{39}{3}}{\binom{52}{5}}$
**Definition:** A histogram is a way to present the probability distribution of a discrete random variable.

Example: Draw the probability distribution $X$.

<table>
<thead>
<tr>
<th>X</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>prob.</td>
<td>0.2</td>
<td>0.1</td>
<td>0.15</td>
<td>0.4</td>
<td>0.15</td>
</tr>
</tbody>
</table>

Base = 1
height = prob.
Base of the rest is centered on the x-value.
Example: The following histogram is only missing the rectangle at $x = 7$.

A) Find $P(X = 7) = .15$

B) Give the probability distribution for $X$.

C) Find $P(2 \leq X < 6) = .1 + .25 + .1$
Example: A task consists of drawing a ball from a box containing 4 red, 3 green, and 1 white and replacing it after the color is noted. Let the random variable $X$ be the number of green balls drawn when this task is repeated 6 times.

$$n = 6 \quad p = \frac{3}{6}$$

A) Compute $P(X = 2)$

$$= \binom{6}{2} \cdot \left(\frac{3}{6}\right)^2 \left(\frac{3}{6}\right)^4 = bpdf(6, \frac{3}{6}, 2)$$

B) Compute $P(X < 5)$

$$= bcdf(6, \frac{3}{6}, 4)$$

Definition: Given a sequence of $n$ Bernoulli trials with the probability of success $p$ and the probability of failure of $q$, the binomial distribution, for $k = 0, \ldots, n$, is given by

$$P(X = k) = C(n, k)p^k q^{n-k}$$
Example: A cookie company wants to check the consistency of the number of raisins in its oatmeal raisin cookies. A few cookies from each batch are selected and the number of raisins are counted. After several days, the following results were found.

<table>
<thead>
<tr>
<th>number of cookies</th>
<th>3</th>
<th>5</th>
<th>6</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>number of raisins</td>
<td>2</td>
<td>7</td>
<td>9</td>
<td>10</td>
</tr>
</tbody>
</table>

Define the random variable and give the probability distribution.

\[
X = \text{# of Raisins in each cookie.}
\]

\[
\begin{array}{c|cccc}
X & 2 & 7 & 9 & 10 \\
\hline
p(x) & \frac{3}{22} & \frac{5}{22} & \frac{6}{22} & \frac{8}{22}
\end{array}
\]