Section 2.1:

An experiment is to flip a coin. How many outcomes are possible if the coin is flipped

A) Twice. \( S = \{ \text{HH, HT, TH, TT} \} \)

\[ 4 = 2 \cdot 2 \]

B) Three times. \( S = \{ \text{HHH, HHT, HTT, TTH, THH, THT, TTH, TTT} \} \)

\[ 8 = 2 \cdot 2 \cdot 2 \]

C) Five times.

\( \frac{2 \cdot 2 \cdot 2 \cdot 2 \cdot 2}{2 \cdot 2 \cdot 2} = 2^5 \)

Example: An experiment is to draw two letters in succession from a box that has an A, B, and C. How many outcomes are there?

\[
\begin{array}{ccc}
\text{With Reps} & \text{No Reps} \\
\text{AB} & \text{AB} & \text{AC} \\
\text{BA} & \text{BA} & \text{CA} \\
\text{CA} & \text{BC} & \text{CB} \\
\text{AB} & \text{BB} & \text{CC} \\
\end{array}
\]

\[ 9 = 3 \cdot 3 \]

\[ 6 = 3 \cdot 2 \]
Definition: An outcome consists of $k$ successive selections with $n_i$ choices for the $i$-th selection. The total number of outcomes is

$$n_1 \cdot n_2 \cdot n_3 \cdot \ldots \cdot n_k$$
Example: There are 6 roads from town A to town B and 7 roads from town B to town C. How many ways can you go from town A to town C?

\[
\begin{array}{c}
\frac{6 \cdot 7}{A \rightarrow B} = 42 \\
\frac{B \rightarrow C}
\end{array}
\]

Example: How many ways can you select a president, vice-president and secretary from a group of 10 people?

\[
10 \cdot 9 \cdot 8
\]
**Definition:** A factorial, \( n! \), is the product of integers from \( n \) down to 1. For example: \( 5! = 5 \times 4 \times 3 \times 2 \times 1 \).

By definition, \( 0! = 1 \)

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**Example:** Compute the following.

A) \( 10! = 3628800 \)  

B) \( 14! = 87 178291200 \)
Example: How many three digit numbers can be formed from the digits: 2, 3, 4, 5, 6, 7, 8?

A) No restrictions. 7 • 7 • 7

B) The number is even. 7 • 7 • 4

C) The digits are even. 4 • 4 • 4

D) The number is even and no digit is repeated. 6 • 5 • 4
Example: Five boys and five girls are to be seated in a row. Find how many ways can this be done if

A) no restrictions.

\[ \frac{10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{10!} = 10! \]

B) they alternate seats.

\[ 2 \cdot \left( \frac{5 \cdot 5 \cdot 4 \cdot 4 \cdot 3 \cdot 3 \cdot 2 \cdot 2 \cdot 1 \cdot 1}{b \; g \; b \; g \; b \; g \; b \; g \; b \; g \; g \; b \; g \; b \; g \; b \; g \; b \; g \; b} \right) = 2 \cdot 5! \cdot 5! \]

\[ \frac{10 \cdot 5 \cdot 4 \cdot 3 \cdot 3 \cdot 2 \cdot 2 \cdot 1 \cdot 1}{10!} = 10! \]
C) girls sit together and boys sit together.

\[ 2 \cdot \left( \frac{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{b \quad g} \right) \]

D) girls sit together.

\[ 6 \cdot \left( \frac{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{g} \right) \]
E) Sue, Jill, or Sarah are seated in the end seats.

\[
\begin{array}{cccccccc}
| & 3 & \cdot & 8 & \cdot & 7 & \cdot & 6 & \\
\hline
& 5 & \cdot & 4 & \cdot & 3 & \cdot & 2 & \cdot 1 \\
\end{array}
\]
Example: Four couples are to be seated in a row. How many ways can this be done if the couples are to be seated together?

$$\left( \frac{4 \cdot 3 \cdot 2 \cdot 1}{2 \cdot 2 \cdot 2} \right) \cdot 2 \cdot 2 \cdot 2 \cdot 2$$

$$8 \cdot 1 \cdot 6 \cdot 1 \cdot 4 \cdot 1 \cdot 2 \cdot 1$$
Example: How many 3 digit numbers can be formed so that

A) none of the digits are a 7. \[ \underline{8 \cdot 9 \cdot 9} \]

B) Exactly one digit is a 7.
\[ \frac{1 \cdot 9 \cdot 9}{\uparrow} + \frac{8 \cdot 1 \cdot 9}{\uparrow} + \frac{8 \cdot 9 \cdot 1}{\uparrow} \]

C) Exactly two digits are a 7.
\[ \frac{1 \cdot 1 \cdot 9}{\uparrow \uparrow} + \frac{1 \cdot 9 \cdot 1}{\uparrow \uparrow} + \frac{8 \cdot 1 \cdot 1}{\uparrow \uparrow} \]

D) Exactly three digits are a 7.

E) no digit is repeated and the number is even.
\[ \underline{9 \cdot 8 \cdot 1} + \underline{8 \cdot 8 \cdot 4} \]

\[ \frac{9 \cdot 8 \cdot 1}{\underline{200}} + \frac{8 \cdot 8 \cdot 4}{\underline{2,416,6}} \]
Example: How many 5 digit numbers have at least one digit being a 7?

\[
\overline{0,1,2,3,4,5,6,7,8,9}
\]

\( \Rightarrow \) one or more sevens

Total - \( \text{don't want} \) = want.

\( \Rightarrow \) have no sevens.

\[
9 \cdot 10 \cdot 10 \cdot 10 \cdot 10 - 9 \cdot 9 \cdot 9 \cdot 9 \cdot 9
\]
Example: An ATM code contains 4 digits. How many codes are possible if the bank will not allow the codes to have all the same digits?

\[
\begin{align*}
10 \cdot 9 \cdot 8 \cdot 7 & \quad \text{all digits diff.} \\
6 \cdot \left( \frac{10 \cdot 1 \cdot 9 \cdot 8}{r} \right) & \quad \text{one pair of same digits} \\
6 \cdot \left( \frac{10 \cdot 1 \cdot 9 \cdot 1}{r} \right) & \quad \text{2 pairs of same digits} \\
4 \cdot \left( \frac{10 \cdot 1 \cdot 1 \cdot 9}{r} \right) & \quad \text{3 same digits}
\end{align*}
\]

Total - don't count all same digit.

\[
10 \cdot 10 \cdot 10 \cdot 10 - 10 = 10^4 - 10
\]
Example: A computer code is to be constructed with either 5 letters or 2 letters followed by three digits. How many codes are possible if

A) there are no restrictions.

$$26 \cdot 26 \cdot 26 \cdot 26 \cdot 26 + 26 \cdot 26 \cdot 10 \cdot 10 \cdot 10$$

B) no letters may be repeated in the code.

$$26 \cdot 25 \cdot 24 \cdot 23 \cdot 22 + 26 \cdot 25 \cdot 10 \cdot 10 \cdot 10$$
Example: Serial numbers assigned to a bicycle by a manufacture have a first symbol of J, H, or T to indicate the plant in which made, followed by 01, 02, 03,..., or 12 to indicate the month in which made, followed by four digits. How many different serial numbers are possible?

\[3 \cdot 12 \cdot 10 \cdot 10 \cdot 10 \cdot 10\]