Section 7.1: Area between Curves

Consider the continuous functions $f(x)$ and $g(x)$ with the property on the interval $[a, b]$ that both are above the x-axis and $f(x) \geq g(x)$. Write down the computation that will give the area bounded between these functions on this interval.

\[
\int_{a}^{b} f(x) \, dx - \int_{a}^{b} g(x) \, dx = \int_{a}^{b} \text{top - bottom} \, dx
\]
For the next graphs, set-up the integral(s) that will give the area that is bounded between \( f(x) \) and \( g(x) \) on the interval \([a, b]\).

\[
\int_{a}^{b} (f(x) - g(x)) \, dx
\]

\[
\int_{a}^{c} f(x) - g(x) \, dx + \int_{c}^{b} g(x) - f(x) \, dx
\]
1. Find the area that is bounded by these curves.

\[ y = x + 3 \]
\[ y = x^2 - 9 \]

\[ x^2 - x - 12 = 0 \]
\[ (x-4)(x+3) = 0 \]
\[ x = 4 \quad x = -3 \]

\[ \int_{-3}^{4} (x+3) - (x^2 - 9) \, dx = \int_{-3}^{4} x^2 - x^2 + 9 \, dx \]

\[ \int_{-3}^{4} x - x^2 + 12 \, dx = \frac{x^2}{2} - \frac{x^3}{3} + 12x \bigg|_{-3}^{4} \]

\[ = \frac{16}{2} - 48 + 48 - \left( \frac{9}{2} - \frac{(-3)^3}{3} - 36 \right) \]

\[ = \frac{343}{6} \]
2. Find the area that is bounded by these curves.

\[ x = y^2 \]
\[ x = 2y^2 - 4 \]

\[ 2y^2 - y = y^2 \]
\[ y^2 = y \]
\[ y = \pm 1 \]

\[ x = 2y^2 - y \]
\[ x + y = 2y^2 \]
\[ \frac{x + y}{2} = y^2 \]
\[ y = \pm \sqrt{\frac{x + y}{2}} \]

\[ \int_{-1}^{1} \left( \sqrt{\frac{x + y}{2}} - \sqrt{\frac{x - y}{2}} \right) dx \]

Top part:
\[ \int_{0}^{1} \sqrt{\frac{x + y}{2}} - \sqrt{x} \, dx \]
2. Find the area that is bounded by these curves.

\[ x = y^2 \]
\[ x = 2y^2 - 4 \]

\[ 2y^2 - y = y^2 \]
\[ y^2 = y \]
\[ y = \pm \frac{1}{2} \]

\[ \int_{-2}^{2} y^2 - (2y^2 - 4) \, dy = \ldots = \frac{32}{3} \]
3. Find the area that is bounded by these curves.

\[
y = e^{-3x}
y = e^x
\begin{cases}
x = -2 \\
x = 1
\end{cases}
\]

\[-2 \leq x \leq 1\]

\[
\int_{-2}^{1} e^{-3x} - e^x \, dx + \int_{0}^{1} e^x - e^{-3x} \, dx
\]

\[
= \cdots = 134.6798
\]
4. Set up the integral(s) that will give area that is bounded by these curves.

\[ x = y^2 - 4y \]
\[ y = 0.5x \]
\[ y = 3 \]
\[ y = -2 \]

\[ \text{Integral} \]
\[ -2 \leq y \leq 3 \]

\[ \int_{0}^{3} 2y - (y^2 - 4y) \, dy + \int_{-2}^{0} y^2 - 4y - 2y \, dy \]

\[ = \frac{98}{3} \]
5. Set up the integral(s) that will give area that is bounded by these curves.

\[ y = \sin(x) \]
\[ y = \cos(2x) \]

\[ x = \pi \]
\[ x = 0.5\pi \]

\[ \int_{\frac{\pi}{6}}^{\pi} \left( \cos(2x) - \sin(x) \right) \, dx + \int_{\frac{\pi}{6}}^{\pi} \cos(2x) - \sin(x) \, dx \]

\[ = -1 + \frac{3\sqrt{3}}{2} \]
6. Set up the integral(s) that will give area that is bounded by these curves.

\[ x = |y - 1| \]
\[ x = y^2 - 3 \]
\[ y \geq 0 \]

\[ y - 1 = y^2 - 3 \]
\[ 0 = y^2 - y - 2 \]
\[ 0 = (y-2)(y+1) \]
\[ y = 2 \quad y = -1 \]

\[ x = \begin{cases} 
  y - 1, & y \geq 1 \\
  -(y - 1), & y < 1 
\end{cases} \]

\[ x = y - 1 \quad \therefore x = y - 1 \]

\[ \int_{0}^{1} -(y - 1) - (y^2 - 3) \, dy + \int_{1}^{2} y - 1 - (y^2 - 3) \, dy \]

\[ = \ldots = \frac{13}{3} \]