Spring 2012 Math 151

Week in Review # 1
sections: Review, Appendix D, 1.1

1. Find the domain of these functions.

(a) \( f(x) = \frac{x + 1}{x^{7/3} - 3x^{4/3} - 10x^{1/3}} = \frac{x+1}{x^{\frac{1}{3}} \left( x^{\frac{1}{3}} - 3x^{\frac{1}{3}} - 10^{\frac{1}{3}} \right)} = \frac{x+1}{\sqrt[3]{x} \left( x - 3x^{\frac{1}{3}} - 10^{\frac{1}{3}} \right)} = \frac{x+1}{\sqrt[3]{x} \left( x - 5 \right) \left( x + 2 \right)} \)

Domain: \((-\infty, -2) \cup (-2, 0) \cup (0, 5) \cup (5, \infty)\)
1. Find the domain of these functions.

(b) \( g(x) = \frac{\sqrt{x^2 - 4}}{\sqrt{x + 5}} \)

\[
\sqrt{x^2 - 4} = \sqrt{(x+2)(x-2)}
\]

\[
\sqrt{x+5}
\]

Domain: \((-\infty, -2] \cup [2, \infty)\)
2. If \( \tan(\theta) = \frac{9}{12} \) and \( \theta \) is in Quadrant III, find the exact values of

\[
\begin{align*}
\sin(\theta) &= -\frac{\sqrt{9^2 + (-12)^2}}{15} \\
\csc(\theta) &= -\frac{15}{9} \\
\cos(\theta) &= -\frac{12}{15} \\
\cot(\theta) &= -\frac{12}{9} \\
\sec(\theta) &= -\frac{15}{12} \\
\end{align*}
\]
Trig. Identities

\[
\begin{align*}
\sin(2x) &= 2\sin(x)\cos(x) \\
\cos(2x) &= 2\cos^2(x) - 1 \\
\cos(x + y) &= \cos(x)\cos(y) - \sin(x)\sin(y) \\
\cos(x - y) &= \cos(x)\cos(y) + \sin(x)\sin(y) \\
\sin(x + y) &= \sin(x)\cos(y) + \cos(x)\sin(y) \\
\sin(x - y) &= \sin(x)\cos(y) - \cos(x)\sin(y)
\end{align*}
\]

Law of Sines

\[
\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}
\]

Law of Cosines

\[
a^2 = b^2 + c^2 - 2bc\cos A
\]
3. If \( \sin(x) = \frac{1}{6} \) and \( \sec(y) = \frac{17}{15} \), where \( x \) and \( y \) lie between 0 and \( \frac{\pi}{2} \), evaluate the expression using trigonometric identities.

(a) \( \sin(2x) = 2 \sin(x) \cos(x) = 2 \left( \frac{1}{6} \right) \cdot \frac{\sqrt{35}}{6} = \frac{\sqrt{35}}{18} = \frac{2 \sqrt{35}}{36} \)
(b) \( \cos(x + y) = \cos(x) \cos(y) - \sin(x) \sin(y) \)
\[
= \frac{\sqrt{35}}{6} \cdot \frac{15}{17} - \frac{1}{6} \cdot \frac{8}{17}
\]

(c) \( \sin(x - y) = \sin(x) \cos(y) - \cos(x) \sin(y) \)
\[
= \frac{1}{6} \cdot \frac{15}{17} - \frac{\sqrt{35}}{6} \cdot \frac{8}{17}
\]
4. The triangle below has the following values: \( c = 4 \), \( a = 5 \) and \( B = 25^\circ \). Find \( b \).

\[
\frac{b}{\sin B} = \frac{a}{\sin A} = \frac{c}{\sin C}
\]

\[
b^2 = a^2 + c^2 - 2ac \cos B
\]

\[
b^2 = 5^2 + 4^2 - 2(5)(4) \cos (25^\circ)
\]

\[
b^2 = 41.747688
\]

\[
b = 2.08789
\]
5. Solve for $x$ where $0 \leq x \leq 2\pi$.

(a) $2\cos^2(x) - \cos(x) - 1 = 0$

$$2A^2 - A - 1 = 0$$

$$2A + 1 = 0$$
$$A = -\frac{1}{2}$$

$$2A = -1$$
$$A = 1$$

$$(2A + 1)(A - 1) = 0$$

$$\cos(x) = -\frac{1}{2}$$

$x = \frac{2\pi}{3}, \frac{4\pi}{3}$

$\cos(x) = 1$

$x = 0, 2\pi$

$x = \theta, \frac{2\pi}{3}, \frac{4\pi}{3}, 2\pi$
<table>
<thead>
<tr>
<th>$\theta$</th>
<th>$\sin \theta$</th>
<th>$\cos \theta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$0^\circ$</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>$30^\circ$</td>
<td>$\frac{\sqrt{3}}{2}$</td>
<td>$\frac{\sqrt{1}}{2}$</td>
</tr>
<tr>
<td>$45^\circ$</td>
<td>$\frac{\sqrt{2}}{2}$</td>
<td>$\frac{\sqrt{2}}{2}$</td>
</tr>
<tr>
<td>$60^\circ$</td>
<td>$\frac{\sqrt{3}}{2}$</td>
<td>$\frac{1}{2}$</td>
</tr>
<tr>
<td>$90^\circ$</td>
<td>1</td>
<td>0</td>
</tr>
</tbody>
</table>
(b) \( \sin(x) \cos(x) = \frac{1}{4} \)

\[ 2 \sin(x) \cos(x) = \frac{1}{2} \]

\[ \sin(2x) = \frac{1}{2} \]

\[ 2x = \frac{\pi}{6} \]

\[ 2x = \frac{5\pi}{6} \]

\[ x = \frac{\pi}{12} \]

\[ x = \frac{5\pi}{12} \]

\[ x = \frac{\pi}{12} + \pi = \frac{13\pi}{12} \]

\[ x = \frac{5\pi}{12} + \pi = \frac{17\pi}{12} \]
Section 1.1

6. Given $A(1, 6)$ and $B(5, -3)$, find the vector $\overrightarrow{BA}$.

$$\overrightarrow{BA} = \langle a_1 - b_1, a_2 - b_2 \rangle$$

$$= \langle 1 - 5, 6 - (-3) \rangle$$

$$= \langle -4, 9 \rangle$$

$$\overrightarrow{AB} = \langle 4, -9 \rangle$$
7. Given $a = 2i + 5j$ and $b = (4, 1)$. Find the following.

(a) $|a| = \sqrt{2^2 + 5^2} = \sqrt{4 + 25} = \sqrt{29}$

(b) $3b - 2a = 3(4, 1) - 2(2, 5) = (12, 3) - (4, 10) = (8, -7)$
7. Given \( \mathbf{a} = 2\mathbf{i} + 5\mathbf{j} \) and \( \mathbf{b} = \langle 4, 1 \rangle \). Find the following.

(c) Find scalars \( s \) and \( t \) so that \( s\mathbf{a} + t\mathbf{b} = \mathbf{c} \) where \( \mathbf{c} = \langle 24, -3 \rangle \)

\[
2 \langle 2, 5 \rangle + t \langle 4, 1 \rangle = \langle 24, -3 \rangle
\]

\[
\left\langle \frac{2s+4t}{5s+t} \right\rangle = \langle 24, -3 \rangle
\]

\[
2s + 4t = 24
\]

\[
5s + t = -3
\]

\[
2s + 4(-3 - 5s) = 24
\]

\[
2s - 12 - 20s = 24
\]

\[
-18s = 36
\]

\[
s = -2
\]

\[
t = -3 - 5(-2)
\]

\[
t = 7
\]
7. Given $a = 2i + 5j$ and $b = (4, 1)$. Find the following.

(d) Find the unit vector that is in the same direction of $b$.

$$|b| = \sqrt{4^2 + 1^2} = \sqrt{16 + 1} = \sqrt{17}$$

$$\frac{1}{\sqrt{17}} \langle 4, 1 \rangle = \langle \frac{4}{\sqrt{17}}, \frac{1}{\sqrt{17}} \rangle$$

(e) Find a vector of length 3 in the opposite direction of $b$.

$$-3 \langle \frac{4}{\sqrt{17}}, \frac{1}{\sqrt{17}} \rangle = \langle \frac{-12}{\sqrt{17}}, \frac{-3}{\sqrt{17}} \rangle$$
8. Two forces $T$ and $S$ with magnitudes 4 pounds and 2 pounds act on an object at a point $P$ as shown in the picture. Find the resultant force as well as its magnitude and direction. (Indicate the direction by finding the angle between the vector and the positive x-axis.)

$$R = T + S$$

$$S = \langle 2, 0 \rangle$$

$$T = \langle 4 \cos 60^\circ, 4 \sin 60^\circ \rangle$$

$$= \langle 2, 2\sqrt{3} \rangle$$

$$R = S + T$$

$$= \langle y, 2\sqrt{3} \rangle$$

$$|R| = \sqrt{y^2 + (2\sqrt{3})^2}$$

$$= \sqrt{16 + 12}$$

$$= \sqrt{28}$$

$$\tan \theta = \frac{2\sqrt{3}}{y}$$

$$\theta = \arctan \left( \frac{2\sqrt{3}}{y} \right)$$
9. Two tug boats are towing a large ship into port. The larger tug exerts a force of 4500 pounds on its cable, and the smaller tug exerts a force of 2700 pounds on its cable. If the ship is to travel in a straight line, find the angle $\theta$ that the larger tug must make if the smaller tug makes an angle of 30°.

\[
P = T_L + T_S = \langle 0, \text{ force} \rangle
\]

\[
T_S = \langle 2700 \cos 60, 2700 \sin 60 \rangle
\]
\[
= \langle 1350, 1350\sqrt{3} \rangle
\]
\[
T_S = \langle 2700 \sin 30, 2700 \sin 60 \rangle
\]

\[
T_L = \langle -4500 \sin \theta, 4500 \cos \theta \rangle
\]

\[
-4500 \sin \theta + 1350 = 0
\]

\[
1350 = 4500 \sin \theta
\]

\[
\frac{1350}{4500} = \sin \theta \rightarrow \theta = \arcsin \left( \frac{1350}{4500} \right)
\]

\[
\theta = 17.46°
\]
10. A pilot wishes to set a course so that his ground speed is northeast (N45°E) at 180 mph. The wind is blowing in the direction of S30°E at 40 mph. What course (speed and bearing) should the pilot set in order to achieve his desired ground speed?

\[ \cos 45° = \frac{\sqrt{2}}{2} \]

\[ R = P + w \]
\[ R - w = P \]

\[ |R| = 180 \]
\[ R = \langle 180 \cos 45°, 180 \sin 45° \rangle = \langle 90\sqrt{2}, 90\sqrt{2} \rangle \]

\[ w = \langle 40 \cos 60°, -40 \sin 60° \rangle = \langle 20, -20\sqrt{3} \rangle \]

\[ P = \langle 90\sqrt{2} - 20, 90\sqrt{2} - 20\sqrt{3} \rangle = \langle 90\sqrt{2} - 20, 90\sqrt{2} + 20\sqrt{3} \rangle \]

\[ P = \langle 107.28, 161.92 \rangle \]

speed = \[ |P| = \sqrt{107.28^2 + 161.92^2} = 194.23 \text{ mph} \]

\[ \tan \theta = \frac{107.28}{161.92} \]
\[ \theta = \arctan \left( \frac{107.28}{161.92} \right) = 33.53° \]

bearing \[ N 33.53° E \]