Section L.1: Introduction to Logic

Logic is the science of correct reasoning and making valid conclusions from a collection of statements.

**Definition:** A **statement** is a declarative sentence that is either true or false but not both.

Example: Which of the following would be a statement?

A) Math is fun. **not a statement**
B) 2 + 4 = 6 **yes** 2 + 4 = 7
C) What time is it? **no.**
D) There are 20 students in this classroom. **yes.**
E) Andy Murray is the 2nd best tennis player in the world. **no.**
F) Roger Federer won the French open in 2009. **yes.**

**Definition:** A **compound statement** is a collection of simple statements joined together with **connectives:** "and" (conjunction), "or" (disjunction), or "not" (negation).

Example: 2 + 4 = 6 and 2 * 10 = 8

2 + 4 = 6 or 2 * 10 = 8

**Definition:** A **conjunction** is a statement of the form "p and q" and is written symbolically as \( p \land q \).

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<tr>
<th>p</th>
<th>q</th>
<th>p \land q</th>
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True only if both p and q are true.
**Definition:** A disjunction is a statement of the form "p or q" and is written symbolically as $p \lor q$.

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<th>p</th>
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<th>$p \lor q$</th>
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As long as one is true, $p \lor q$ is true. Inclusive or

**Definition:** A negation is a statement of the form "not p" and is written symbolically as $\sim p$.

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<tr>
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<th>$\sim p$</th>
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Example: Express the following compound statements with symbolic notation.

$\neg p$: The shirt was bought from Target.  
$b\ q$: The shirt is blue.

$\neg r$: The shirt is a short sleeve shirt.

A) The shirt was bought from Target and it was blue.  

$\neg r \land \neg q$

B) The shirt was blue or it had short sleeves.

$q \lor r$

C) The shirt was not short sleeves and it was blue.

$\sim r \land q$

D) The short sleeve blue shirt was not bought at target.

$r \land q \land \sim p$
Example: Use the statements \( r \), \( f \), and \( n \) to express the following compound statements in words.

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\( r \): The truck is red. \\
\( n \): The truck is new. \\
\( f \): The truck is a Ford.

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A) \( n \land r \land \sim f \) \\
The truck is new and is red and is not a Ford.

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The new red truck is not a ford.

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B) \( \sim r \lor f \) \\
The truck is not red or it is a Ford.

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C) \( n \land (r \lor f) \) \\
The new truck is red or a Ford.