Section F.2: Compound Interest

Definition: Suppose the principal, $P$, is invested for $t$ years at a annual interest rate of $r\%$ and interest is compounded $m$ times per year. The future amount, $A$ or $F$, is given by

$$A = P(1 + i)^n = P \left(1 + \frac{r}{m}\right)^{mt}$$

Example: Find the balance of the account if you invest $8600$ for 7 years at a nominal rate of 5% compounded

A) annually.
$$m=1, \quad A = 8600 \left(1 + \frac{0.05}{1}\right)^{1(7)} = 844.26$$

B) semiannually.
$$m=2, \quad A = 8600 \left(1 + \frac{0.05}{2}\right)^{2(7)} = 847.78$$

C) quarterly.
$$m=4, \quad A = 8600 \left(1 + \frac{0.05}{4}\right)^{4(7)} = 849.60$$

D) monthly.
$$m=12, \quad A = 8600 \left(1 + \frac{0.05}{12}\right)^{12(7)} = 850.82$$

E) daily.
$$m=365, \quad A = 8600 \left(1 + \frac{0.05}{365}\right)^{365(7)} = 851.42$$
Example: You want $2000 in an account at the end of 3 years. If the account gets a nominal rate of 5.75% compounded quarterly, how much do you start the account with?

\[ A = P \left(1 + \frac{r}{m}\right)^{mt} \]

\[ r = \frac{A}{\left(1 + \frac{r}{m}\right)^{mt}} = A \left(1 + \frac{r}{m}\right)^{-mt} \]

\[ r = \frac{2000}{\left(1 + \frac{0.0575}{4}\right)^{4(3)}} = \$1685.18 \]

Example: You have the choice of investing money in one of two different accounts. The first account is at Bank A and has a rate of 6.51% compounded semiannually. The second account is at Bank B and has a rate of 6.08% compounded daily. Which account is the better deal?

\[ \frac{A}{100 \left(1 + \frac{0.0651}{2}\right)^{2} - 100} \]

\[ \frac{B}{100 \left(1 + \frac{0.0608}{365}\right)^{365} - 100} \]

\[ = 6.1595\% \]

\[ = 6.2680\% \]

If the account had simple interest for 1 year, \( r_{eff} \) would be the simple interest rate.

Definition: For compound interest, the effective yield, \( r_{eff} \), is given by

\[ r_{eff} = \frac{100 \left(1 + \frac{r}{m}\right)^{m} - 100}{r} \]

\( \text{or} \quad \text{eff}(r, m) \)
Example: You invest $2000 in an account that pays interest compounded monthly. What interest rate do you need to have a balance of $5000 at the end of 3 years.

\[ A = P \left( 1 + \frac{r}{12} \right)^{12t} \]

\[ 5000 = 2000 \left( 1 + \frac{r}{12} \right)^{12 \times 3} \]

\[ \frac{5}{2} = \left( 1 + \frac{r}{12} \right)^{36} \]

\[ \left( \frac{5}{2} \right)^{\frac{1}{36}} = 1 + \frac{r}{12} \]

\[ \left( \frac{5}{2} \right)^{\frac{1}{36}} - 1 = \frac{r}{12} \]

\[ 12 \left( \left( \frac{5}{2} \right)^{\frac{1}{36}} - 1 \right) = r \]

\[ r = 30.935\% \]

Example: A zero coupon bond will mature in 5 years and has a face value of $8000. If the bond has a return of 4.75% compounded annually, how much should you pay for it?

\[ \begin{align*}
N &= 1 \times 5 \\
I &= 4.75 \\
PV &= \underline{-6343.37} \\
PMT &= 0 \\
FV &= 8000 \\
P/Y &= 1 \\
C/Y &= 1
\end{align*} \]

Answer is $6343.37
TVM Solver

The TVM solver that is built function on the TI-83/84 calculators. If you are using the old TI-83 press \[2\text{nd}\ x^{-1}\] and then press \[\text{ENTER}\], otherwise press the \[\text{APPS}\] and the select the \textbf{Finance} application and press enter. Here are the variables that are used in the TVM Solver.

\[
\begin{align*}
N &= m \times t \text{ which is the total number of periods} \\
I\% &= \text{The interest rate per year as a percentage.} \\
PV &= \text{The present value(starting value) of the account.} \\
PMT &= \text{This is the payment that is made each period.} \\
FV &= \text{The future value(end value) of the account.} \\
P/Y &= \text{The number of payments per year.} \\
C/Y &= \text{The number of compoundings per year.}
\end{align*}
\]

For this class, \(P/Y\) and \(C/Y\) are equal and \(\text{PMT:END BEGIN}\) should be set to END.