Section F.1: Simple Interest and Discounts

**Definition:** If the principal, \( P \), is invested for a time period of \( t \) at a **simple interest rate** of \( r\% \) (for that period) then the interest earned at the end of the time period is given by

\[
I = Prt
\]

The future value, \( A \) or \( F \) of the investment at the end of the period is

\[
A = P + I = P(1 + rt)
\]

Example: You invest $500 at an **annual** simple interest rate of 4% for 6 years. How much interest did you earn? What is the balance at the end?

\[
I = Prt = 500 \times (0.04) \times (6) = \$120
\]

\[
A = 500 + 120 = \$620
\]

Example: You invest $1000 at a **monthly** simple interest rate of 6.5% for 2 years. How much interest did you earn? What is the balance at the end?

\[
I = Prt = (1000)(0.065)(24) = \$1560
\]

\[
A = 1000 + 1560 = \$2560
\]
Example: You invest $2000 for 8 months and at the end of this time period you have earned $400 of interest. What is the annual simple interest rate? monthly simple interest rate?

\[
I = Prt
\]

\[
\frac{\text{Annual}}{400} = 2000 \cdot (r) \left( \frac{8}{12} \right)
\]

\[
r = 0.3 \quad \Rightarrow \quad 30\%
\]

\[
\frac{\text{Monthly}}{400} = 2000 \cdot (r) \cdot 8
\]

\[
r = 0.025 \quad \Rightarrow \quad 2.5\%
\]

\[A = P \left( 1 + rt \right)\]

\[
2400 = 2000 \cdot (1 + r \cdot 8)
\]

Example: You want to have a total of $3000 at the end of 3 years. If the account has an annual simple interest rate of 4.5%, what amount do you have to invest to meet your goal?

\[A = P \left( 1 + rt \right)\]

\[
3000 = P \left( 1 + 0.045 (3) \right)
\]

\[
P = \frac{3000}{1 + 0.045(3)} = \$2643.17
\]
Definition: The **discount** $D$, on a discounted loan of $M$ dollars at a simple interest rate of $r\%$ for the time period $t$ is

$$D = Mrt$$

The **Proceeds** $P$, of the loan is the actual amount the borrower receives from the loan is given by

$$P = M - D = M(1 - rt)$$

Example: John will pay back a $10,000 loan at the end of 15 months. This loan has an annual simple discount rate of 7%. What is the discount on the loan and How much money does John actually receive from the bank?

$$D = Mrt = 10000 \times \left(\frac{0.07}{12}\right) = \$875$$

$$P = M - D = 10000 - 875 = \$9125$$

Example: Susan needs $1,200 right now. She has agreed to take out a discount loan from bank and repay it in 2 years. If the annual discount rate is 8%, compute the maturity value of the loan.

$$P = 1200$$

$$P = M(1 - rt)$$

$$1200 = M \left(1 - \frac{0.08}{2}\right)$$

$$M = \$1428.57$$

Example: What is the effective rate of interest for the loan in the previous example?

$$r_{eff} = \frac{D}{P} = \frac{228.57}{1200 \times 2} \approx 0.0952375$$

$$r_{eff} = 9.52375\%$$
Definition: The effective interest rate, effective yield, on a discount loan of length $t$ years with an annual discount rate is given by

$$r_{eff} = \frac{D}{P_t} = \frac{\frac{m rt}{m(1-rt)^t}}{1-rt} = \frac{r}{1-r^t}$$

Example: What is effective yield on a discount loan with an annual simple interest rate of 4% when the loan is due in 4 months?

$$r_{eff} = \frac{0.04}{1-0.04 \left(\frac{4}{12}\right)} = 0.040541\%$$

\[\text{4.0541}\%\]