Section 3.4: The Normal Distribution

To find probability with a continuous random variable, \( X \), we use a probability density function, \( f(x) \). This function has the properties: 1) \( f(x) \geq 0 \) for all values of \( X \) and 2) the area under the graph of \( f(x) \) is equal to 1 (on the values of \( X \)).

Normal Curve:

This distribution is always centered around the mean. The standard deviation regulates how high of a peak the curve will have. The formula for this curve is

\[
f(x) = \frac{1}{\sigma \sqrt{2\pi}} \cdot e^{-\frac{(x-\mu)^2}{2\sigma^2}}
\]

Shade the area under the normal curve that represents these probabilities.

\[ P(X > a) = P(x \geq a) \]

\[ P(c < X < d) = P(c \leq x \leq d) \]

\[ P(X = a) = 0 \]
Definition: The **standard normal curve** is the normal curve with \( \mu = 0 \) and \( \sigma = 1 \). The random variable for the standard normal curve is \( Z \).

Note: This is the curve that is used to create the normal distribution charts found in the back of the math book. These charts **will not** be used in this course. To convert to the standard normal curve use the formula

\[
Z = \frac{x - \mu}{\sigma}.
\]

<table>
<thead>
<tr>
<th>( z )</th>
<th>.00</th>
<th>.01</th>
<th>.02</th>
<th>.03</th>
<th>.04</th>
<th>.05</th>
<th>.06</th>
<th>.07</th>
<th>.08</th>
<th>.09</th>
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<tbody>
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<td>.5080</td>
<td>.5120</td>
<td>.5160</td>
<td>.5199</td>
<td>.5238</td>
<td>.5275</td>
<td>.5319</td>
<td>.5359</td>
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<td>.5438</td>
<td>.5478</td>
<td>.5517</td>
<td>.5557</td>
<td>.5596</td>
<td>.5636</td>
<td>.5675</td>
<td>.5714</td>
<td>.5753</td>
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<tr>
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<td>.5832</td>
<td>.5871</td>
<td>.5910</td>
<td>.5948</td>
<td>.5987</td>
<td>.6026</td>
<td>.6064</td>
<td>.6103</td>
<td>.6141</td>
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<td>.6293</td>
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<tr>
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<td>.6628</td>
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<td>.6700</td>
<td>.6736</td>
<td>.6772</td>
<td>.6808</td>
<td>.6844</td>
<td>.6879</td>
</tr>
</tbody>
</table>

**Calculator commands**

The TI-84 has built-in commands that we will use to work with normal distributions. They can be found in the Distribution menu by pressing \( \text{2nd} \{ \text{VARS} \} \). **Note:** to enter \( 1E99 \) press \( \text{1} \{ \text{EE} \} \text{9} \). The EE represents scientific notation and the \( \text{EE} \) is found by pressing \( \text{2nd} \{ \} \).

\[ \text{normalcdf(lower, upper, } \mu, \sigma) \] computes the probability that a continuous R.V. \( X \) is between the lower bound and the upper bound.

<table>
<thead>
<tr>
<th>Calculate</th>
<th>Lower</th>
<th>Upper</th>
</tr>
</thead>
<tbody>
<tr>
<td>( P(X &lt; a) )</td>
<td>(-1E99)</td>
<td>( a )</td>
</tr>
<tr>
<td>( P(X &gt; a) )</td>
<td>( a )</td>
<td>( 1E99 )</td>
</tr>
<tr>
<td>( P(a &lt; X &lt; b) )</td>
<td>( a )</td>
<td>( b )</td>
</tr>
</tbody>
</table>

\[ \text{invnorm(area, } \mu, \sigma) \] will return a value \( A \) that satisfies the equation \( P(X < A) = \text{area} \).
Example: Let $X$ be a continuous random variable that is normally distributed with $\mu = 53$ and $\sigma = 4$. Compute the following.

A) $P(45 < X < 58)$

\[
\text{normalcdf} (45, 58, 53, 4) = .8716
\]

B) $P(X < 56)$

\[
\text{normalcdf} (-1E99, 56, 53, 4) = .7731
\]

C) $P(X > 38)$

\[
\text{normalcdf} (38, 1E99, 53, 4) = .955912
\]

Example: Compute $P(0.5 < Z < 2)$

\[
\text{normalcdf} (.5, 2, 0, 1) = .285787
\]

Example: The length of what are considered "one-inch" bolts is found to be a normally distributed random variable with a mean of 1.001 inches and a standard deviation 0.002 inches. If a bolt measures more than two standard deviations from the mean, it is rejected as not meeting factory tolerances. What percentage of the bolts will the factory reject?

\[
1 - \text{normalcdf}(.997, 1.005, 1.001, 0.002)
\]

\[
1 - .954999876 = .045055 \rightarrow \text{Reject 4.55%}
\]
Example: A particular brand of dishwasher has a life expectancy that is estimated to be normally distributed, with a mean of 10 years, 8 months and a standard deviation of 1 year, 2 months. Suppose that such dishwashers are guaranteed to last 9 years. Of every 250 sold, how many will fail to last through the guarantee period?

\[ \text{normalcdf} (-1.52, 108, 128, 14) = .07656 \]

\[ 250 (.07656) = 19.1409 \]

So approximate 19.

Example: The weight of infants is normally distributed with a mean of 7.4lbs and a standard deviation of 1.2lbs. Fifty infants are selected at random, find the probability that at most 10 of them weigh between 7 and 8 pounds.

\[ \text{normalcdf}(7, 8, 7.4, 1.2) = .32202 \]

\[ \text{binomcdf}(50, .32202, 10) = \]
Example: Let $X$ be a normally distributed random variable with $\mu = 53$ and $\sigma = 4$.

A) Find $A$ such that $P(X < A) = 0.350$

\[ A = \text{invnorm}(0.350, 53, 4) = 51.5233 \]

B) Find $B$ such that $P(X > B) = 0.525$

\[ B = \text{invnorm}(1-0.525, 53, 4) = 52.7492 \]

Example: Find $J$ such that $P(-J < Z < J) = 0.78$

Because of symmetry, both tails have the same area, i.e.,

\[ \frac{0.22}{2} = 0.11 \]

\[ J = \text{invnorm}(0.11 + 0.78, 0, 1) = 1.2265 \]
Example: A prof is going to grade an exam on a bell curve. The prof decided that the top 10% of the bell curve will be an A, the next 15% a B, the next 30% a C, and the next 20% a D. If the mean for the class is 70 and the standard deviation is 18, find the lowest grade on the exam that will get you A and the lowest grade on the exam that will get you a B.

\[ A = \text{invnorm}(0.90, 70, 18) = 93.0679 \]

\[ B = \text{invnorm}(0.75, 70, 18) = 82.14 \]