Section 2.3: Probability Applications to Counting Principles

Example: A type of license plate has three letters followed by two digits. If one of these plates is select at random, find the probability that

A) The license plate has all vowels.

\[ P(E) = \frac{n(E)}{n(S)} = \frac{5 \cdot 5 \cdot 5 \cdot 10 \cdot 10}{26 \cdot 26 \cdot 26 \cdot 10 \cdot 10} \]

B) The letters are selected from the set \{a, b, c, d, e\}, the digit zero is not used and no letter is repeated.

\[ \frac{5 \cdot 4 \cdot 3 \cdot 9 \cdot 9}{26^3 \cdot 10^2} \]

Example: A box contains 7 red, 6 green, 5 black, and 3 purple balls. What is the probability that in a sample of 6 that

A) all the balls are the same color.

\[ \frac{C(7,6) + C(4,6)}{C(21,6)} \]

B) exactly three red and at least 2 purple balls were chosen.

\[ \frac{R}{P} \quad \text{other} \]

\[
\begin{array}{ccc}
3 & 2 & 1 \\
3 & 3 & 0
\end{array}
\]

\[ \frac{C(7,3) \cdot C(3,2) \cdot C(11,1) + C(7,3) \cdot C(3,3)}{C(21,6)} \]
Example: An exam consists of 25 questions in which 12 of them must be answered. What is the probability that a student answered exactly 2 of the first 5 questions and at least 4 of the last 5 questions?

\[
C(5, 2) \cdot C(15, 6) \cdot C(5, 4) + C(5, 2) \cdot C(15, 5) \cdot C(5, 5)
\]

\[
C(25, 12)
\]

Example: Box A and Box B contain the following items (see below). Three items are to be drawn from box A and placed into Box B and then one item is to be drawn from Box B. Box A Box B
\[
\begin{array}{c|c}
25 \text{ red} & 2 \text{ red} \\
15 \text{ blue} & 1 \text{ blue}
\end{array}
\]

Find the probability of the last item is blue.

\[
\frac{115}{494} \cdot \frac{1}{6} + \frac{225}{494} \cdot \frac{2}{6} + \frac{525}{1976} \cdot \frac{3}{6} + \frac{2}{152} \cdot \frac{4}{6}
\]
Example: Find the probability that in a group of 10 people that all were born on a different days of the year.

\[
\begin{align*}
\frac{365 \cdot 364 \cdot 363 \cdot 362 \cdot 361 \cdot 360 \cdot 359 \cdot 358 \cdot 357 \cdot 356}{365^{10}} &= \frac{P(365,10)}{365^{10}}
\end{align*}
\]

Example: Find the probability that at least two people in a group of 10 were born on the same day.

\[
\begin{align*}
1 - \frac{P(365,10)}{365^{10}}
\end{align*}
\]

Example: Find the probability that in a group of 15 people, that at least two people were born in the same month. Assume that months are equally likely.
Example: You have 5 A’s, 2 B’s, and 4 C’s. If all of these letters are mixed up and then placed in a row, what is the probability that identical letters are grouped together?

\[ \frac{3!}{5!2!4!} \]

Example: Two freshmen, 16 sophomores and 12 juniors all apply to attend a national conference. If five of these students are selected to attend the conference, what is the probability that exactly three of the students are juniors if we know that David and Susan, who are both sophomores, are two of the students picked to attend the conference.

\[ \frac{C(12, 3)}{C(17, 3)} \]

\[ \uparrow \text{ conditional prob.} \]
Example: A 6 sided die is rolled 5 times. What is the probability that exactly 3 of the rolls are a 4.

\[ \left( \frac{1}{6} \cdot \frac{1}{6} \cdot \frac{1}{6} \cdot \frac{5}{6} \cdot \frac{5}{6} \right) \cdot \binom{5}{3} \]

\[ \frac{\binom{5}{3} \cdot 1 \cdot 1 \cdot 1 \cdot 5 \cdot 5}{6 \cdot 6 \cdot 6 \cdot 6 \cdot 6} \]

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<th># of people in the group</th>
<th>prob that at least 2 have same birthday</th>
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