Section 1.1

1. Given \( A(1,6) \) and \( B(5, -3) \), find the vector \( \overrightarrow{BA} \).

\[
\overrightarrow{BA} = \begin{bmatrix} 1 - 5 \cr 6 - (-3) \end{bmatrix} = \begin{bmatrix} -4 \cr 9 \end{bmatrix}
\]

2. Given \( a = 2i + 5j \) and \( b = (4,1) \). Find the following.

(a) \( |a| = \sqrt{2^2 + 5^2} = \sqrt{4 + 25} = 5.92 

(b) \( 3b - 2a = 3 \begin{bmatrix} 4 \cr 1 \end{bmatrix} + \begin{bmatrix} -2 \cr 1 \end{bmatrix} = \begin{bmatrix} 10 \cr 4 \end{bmatrix} \)

(c) Find scalars \( s \) and \( t \) so that \( sa + tb = c \) where \( c = (24, -3) \)

\[
\begin{align*}
\begin{bmatrix} 2 \cr 4 \end{bmatrix} + \begin{bmatrix} 4t \\
5s + t \end{bmatrix} &= \begin{bmatrix} 24 \\
-3 \end{bmatrix} \\
2a + 4t &= 24 \\
5s + t &= -3 \\
\frac{2a + 4t}{5s + t} &= \frac{24}{-3} \\
\frac{10 + 4t}{5s + t} &= \frac{-12}{-3} \\
\frac{18}{5s + t} &= \frac{36}{-3} \\
\end{align*}
\]

(d) Find a vector of length 3 in the opposite direction of \( b \)

\[
|b| = \sqrt{4^2 + 1^2} = \sqrt{17}
\]

Unit vector in the same direction of \( b \) is \( \frac{1}{\sqrt{17}} \begin{bmatrix} -4 \\
1 \end{bmatrix} 
\]

Multiply the unit vector by \( -3 \) to make it length 3 in opposite direction.

\[
\frac{-3}{\sqrt{17}} \begin{bmatrix} -4 \\
1 \end{bmatrix} = \begin{bmatrix} -12 \\
-3 \end{bmatrix}
\]
3. Two tug boats are towing a large ship into port. The larger tug exerts a force of 4500 pounds on its cable, and the smaller tug exerts a force of 2700 pounds on its cable. If the ship is to travel in a straight line, find the angle $\theta$ that the larger tug must make if the smaller tug makes an angle of 30°.

\[
\begin{align*}
F_1 &= \langle 4500 \sin \theta, 4500 \cos \theta \rangle \\
F_2 &= \langle 2700 \sin 30°, 2700 \cos 30° \rangle \\
&= \langle 1350, 1350 \sqrt{3} \rangle \\
F_3 &= \langle 0, |F_3| \rangle \\
F_1 + F_2 &= F_3
\end{align*}
\]

\[
-4500 \sin \theta + 1350 = 0
\]

\[
1350 = 4500 \sin \theta
\]

\[
\frac{1350}{4500} = \sin \theta
\]

\[
\sin \theta = .3 \\
\theta = \sin^{-1}(.3) = 17.46°
\]

4. A pilot wishes to set a course so that his ground speed is northeast (N45°E) at 180 mph. The wind is blowing in the direction of S50°E at 40 mph. What course (speed and bearing) should the pilot set in order to achieve his desired ground speed?

\[
\begin{align*}
\gamma &= \langle 180 \cos 45°, 180 \sin 45° \rangle \\
&= \langle 90\sqrt{2}, 90\sqrt{2} \rangle \\
\omega &= \langle 40 \sin 30°, -40 \cos 30° \rangle \\
&= \langle 20, -20 \sqrt{3} \rangle \\
\rho + \omega &= \gamma
\end{align*}
\]

\[
\begin{align*}
\rho &= \langle 90\sqrt{2}, 90\sqrt{2} \rangle - \langle 20, -20 \sqrt{3} \rangle \\
&= \langle 90\sqrt{2} - 20, 90\sqrt{2} + 20\sqrt{3} \rangle \\
&= \langle 107.279, 161.92 \rangle
\end{align*}
\]

\[
\tan \gamma = \frac{107.279}{161.92} \\
\gamma = \tan^{-1}\left(\frac{107.279}{161.92}\right) = 33.5°
\]

bearing: N33.5°E

\[
\begin{align*}
speed: \quad |\rho| &= \sqrt{(107.279)^2 + (161.92)^2} \\
&= 194.234 \text{ mph}
\end{align*}
\]
Section 1.2

5. Find $a \cdot b$ given the following information:

(a) $a = (-3, 5)$ and $b = (1, 2)$

\[
a \cdot b = (-3)(1) + (5)(2) = -3 + 10 = 7
\]

(b) $|a| = 5$, $|b| = 12$, and the angle between $a$ and $b$ is $60^\circ$.

\[
a \cdot b = |a||b|\cos \theta = 5 \cdot 12 \cdot \cos (60^\circ) = 5 \cdot 12 \cdot \left( \frac{1}{2} \right) = 30
\]

6. Find the angle between the vectors $(3, 2)$ and $(-2, 1)$

\[
|a| = \sqrt{9 + 4} = \sqrt{13}
\]
\[
|b| = \sqrt{4 + 1} = \sqrt{5}
\]

\[
a \cdot b = |a||b|\cos \theta = 3(-2) + 2(1) = \sqrt{13} \cdot \sqrt{5} \cdot \cos \theta = -6 + 2 = \sqrt{65} \cdot \cos \theta
\]

\[
\frac{-4}{\sqrt{65}} = \cos \theta
\]

\[
\theta = \cos^{-1} \left( \frac{-4}{\sqrt{65}} \right) = 119.7^\circ
\]
7. Find the value(s) of $x$ so that the following vectors are orthogonal:

$a = (2x, 5)$ and $b = (x, x - 5)$

\[
\mathbf{a} \cdot \mathbf{b} = 0
\]

\[
2x(x) + 5(x - 5) = 0
\]

\[
2x^2 + 5x - 25 = 0
\]

\[
(2x - 5)(x + 5) = 0
\]

\[
x = \frac{5}{2}, \quad x = -5
\]

8. Find a **unit** vector that is **orthogonal** to $a = 4i + 3j$

\[
|a| = \sqrt{16 + 9} = \sqrt{25} = 5
\]

unit vector $\frac{1}{5} \begin{pmatrix} y, 3 \end{pmatrix} = \left\langle \frac{y}{5}, \frac{3}{5} \right\rangle$

\[
a = \langle a_1, a_2 \rangle
\]

\[
a^\perp = \langle -a_2, a_1 \rangle
\]

\[
\left\langle \begin{pmatrix} -\frac{3}{5}, \frac{4}{5} \end{pmatrix} \right\rangle
\]

also $\left\langle \begin{pmatrix} \frac{3}{5}, -\frac{4}{5} \end{pmatrix} \right\rangle$ is a valid answer
9. Find the scalar and vector projection of \(<-2, 1>\) onto \(<6, 1>\).

\[
\text{Scalar} = \text{comp}_a b = \frac{a \cdot b}{|a|} \\
\text{Vector} = \text{proj}_a b = \frac{a \cdot b}{|a|^2} a
\]

\[
\text{Scalar} = \frac{-12 + 1}{\sqrt{37}} = \frac{-11}{\sqrt{37}} \\
\text{Vector} = \frac{-11}{(\sqrt{37})^2} \langle 6, 1 \rangle = \left\langle \frac{-66}{37}, \frac{-11}{37} \right\rangle
\]

10. Find the value of \(x\) so that vector projection of \(b = \langle x, 7 \rangle\) onto \(a = \langle 1, 4 \rangle\) is \(\langle 5, 20 \rangle\).

\[
\frac{x + 28}{17} \langle 1, 4 \rangle = \langle 5, 20 \rangle \\
\frac{x + 28}{17} = \frac{5}{17} \\
(x + 28) = 5 \\
x + 28 = 85 \\
x = 57
\]
11. Find the distance from the point \((4,0)\) to the line \(y = 2x + 1\).

\[ a = (1, 2) \]
\[ b = (4, 1) \]

\[ |a| = \sqrt{5} \]
\[ |b| = \sqrt{16+1} = \sqrt{17} \]

\[ \text{comp}_a b = \frac{4-2}{\sqrt{5}} = \frac{2}{\sqrt{5}} \]

\[ J^2 + \left( \frac{2}{\sqrt{5}} \right)^2 = (\sqrt{17})^2 \]
\[ J^2 + \frac{4}{5} = 17 \]
\[ J^2 = \frac{85}{5} - \frac{4}{5} = \frac{81}{5} \]
\[ J = \frac{9}{\sqrt{5}} \]

12. A constant force of \( \mathbf{F} = 12\mathbf{i} + 15\mathbf{j} \), magnitude is in Newtons, moves an object along a straight line from the point \((1, 5)\) to the point \((6, 8)\). Find the work done if the distance is measured in meters.

\[ W = \mathbf{F} \cdot \mathbf{D} \]
\[ = 12(5) + 15(3) \]
\[ = 105 \text{ Nm} \]
\[ = 105 \text{ J} \]
13. A crate is pulled on a level surface for a distance of 50m under a constant force of 25N. The force is applied at an angle of 20° with the ground. Find the work done to move the crate.

\[ W = F \cdot D = |F| \cdot |D| \cos \theta \]

\[ = |25| \cdot |50| \cos(20) \]

\[ = 1174.6 \text{ J} \]