1. Use differentials to approximate $\sqrt{11}$.

   a) $\frac{8}{3}$  
   b) $\frac{11}{3}$  
   c) $\frac{23}{6}$

   d) $\frac{21}{6}$  
   e) $\frac{10}{3}$

\[
y = \sqrt{x} \\
dy = \frac{1}{2} x^{-\frac{1}{2}} \, dx \\
dy = \frac{1}{2\sqrt{x}} \, dx \\
dy = \frac{1}{2\sqrt{11}} \cdot 2 = \frac{1}{3}
\]

\[
a = 9 \quad \Delta x = 2 \quad \Delta y = \sqrt{11} + \Delta y \\
\approx \sqrt{9} + \frac{1}{3} = \frac{10}{3}
\]

2. If $f(x) = 3x \cos^2(x^2)$, find $f'(0)$.

   a) 0  
   b) $-3$  
   c) 3  
   d) 1  
   e) $-9$

\[
f'(x) = 3 \cos^2(x^2) + 3x \cdot 2 \cos(x^2) \cdot (-\sin(x^2)) \cdot 2x \\
= 3 \cos^2(x^2) - 12x^2 \cos(x^2) \sin(x^2) \\
f'(0) = 3 \cos^2(0) = 3
\]
3. Compute \( \log_4 2 \)

a) 4  

b) \( \frac{3}{2} \)  

c) \( \frac{1}{2} \)  

d) 2  

e) \( \frac{2}{3} \)

\[
2^x = 2^{2^x} \quad 2^2 = 2^1 \\
2^{2x} = 2 \\
2x = 1 \\
x = \frac{1}{2}
\]

4. Two sides of a triangle are fixed at 4cm and 6cm and the angle between them is increasing at a rate of .02 radians per second. How fast is the area of the triangle increasing when the angle between them is \( \frac{\pi}{6} \)?

a) \( (.12)\sqrt{3} \)  

b) \( \frac{0.2}{6} \)  

c) \( \frac{0.2}{6\sqrt{3}} \)  

d) .12  

e) 12 \sin(.02)

\[
A = \frac{1}{2}bh \\
= \frac{1}{2}(4) \cdot 4 \sin \theta \\
A = 12 \sin \theta \\
\frac{dA}{dt} = 12 \cos \theta \frac{d\theta}{dt}
\]

\[
\frac{dA}{dt} = 12 \cos \left( \frac{\pi}{6} \right) \cdot .02 \frac{d\theta}{sec} \\
= 12 \left( \frac{\sqrt{3}}{2} \right) (.02) = .12\sqrt{3} \text{ cm/sec}
\]
5. Let \( f(x) = (1 + x^2)^{\frac{3}{2}} \). Then \( f''(0) = \)

a) 3 \hspace{1cm} b) 0 \hspace{1cm} c) 6

d) \( \frac{3}{4\sqrt{2}} \) \hspace{1cm} e) \( \frac{3}{4} \)

\[ f' = \frac{3}{2} (1 + x^2)^{\frac{1}{2}} \cdot 2x = 3x (1 + x^2)^{-\frac{1}{2}} \]

\[ f'' = 3 (1 + x^2)^{-\frac{1}{2}} + 3x \frac{1}{2} (1 + x^2)^{-\frac{3}{2}} \cdot 2x \]

\[ f''(0) = 3 \cdot 1 + 0 = 3 \]

6. Solve for \( x \): \( \log(3 - x) + \log(x + 4) = 1 \)

a) \( x = \frac{-1 \pm \sqrt{89}}{2} \) \hspace{1cm} b) \( x = 1 \) only

c) no solution \hspace{1cm} d) \( x = \frac{-1 + \sqrt{89}}{2} \) only

e) \( x = 1 \) or \( x = -2 \)

\[ \log [(3-x)(x+4)] = 1 \]
\[ \log [-x^2-x+12] = 1 \]
\[ 10^1 = -x^2-x+12 \]
\[ x^2 + x - 2 = 0 \]

\[ (x+2)(x-1) = 0 \]
\[ x = -2 \]
\[ x = 1 \]

Check solutions
7. The function \( f(x) = x^3 + 5x - 1 \) is one-to-one. Let \( g = f^{-1} \).

Then \( g'(5) = \)

a) 8 \hspace{1cm} b) \frac{1}{80} \hspace{1cm} c) \frac{8}{25}

d) \frac{1}{8} \hspace{1cm} e) 80

\[
f(g(x)) = x
\]
\[
f'(g(x)) \cdot g'(x) = 1
\]
\[
g'(x) = \frac{1}{f'(g(x))}
\]
\[
f'(x) = 3x^2 + 5
\]
\[
g'(5) = \frac{1}{3 + 5} = \frac{1}{8}
\]

8. Given the curve parametrized by \( x = t^3 - 3t^2 - 9t + 1 \),
\( y = t^3 + 3t^2 - 9t + 1 \), at which point does the curve have a vertical tangent?

a) \((1, -3)\) \hspace{1cm} b) \((6, 12)\) \hspace{1cm} c) \((-10, 6)\)

d) \((-1, 3)\) \hspace{1cm} e) \((1, 1)\)

\[
\frac{dx}{dt} = 3t^2 - 6t - 9
\]
\[
0 = 3(t^2 - 2t - 3)
\]
\[
= 3(t - 3)(t + 1)
\]
\[
t = 3 \quad \text{or} \quad t = -1
\]

\[
\frac{dy}{dt} = 3t^2 + 9t - 9
\]
\[
= 3(t - 1)(t + 3)
\]
\[
t = 1 \quad \text{or} \quad t = -3
\]
9. \[ \lim_{x \to 0} \frac{4 \cos x - 4 + 3 \sin x}{5x} = \]

\[ \frac{\cos \theta - 1}{\theta} \to 0 \quad \text{as} \quad \theta \to 0 \]

\[ \lim_{\theta \to 0} \frac{\sin \theta}{\theta} = 1 \]

a) \( \frac{4}{5} \)  
b) \( -\frac{4}{5} \)  
c) \( \frac{3}{5} \)  
d) 1  
e) 0

\[ \lim_{x \to 0} \frac{4(\cos x - 1)}{5x} + \lim_{x \to 0} \frac{3 \sin x}{5x} \]

\[ 0 + \frac{3}{5} \]

\[ \lim_{x \to 0} \frac{3}{5} \cdot \frac{\sin x}{x} \]

10. Find the slope of the line tangent to the curve given by \( y^2 + xy = 8 \) at the point \((-2, -2)\).

a) \(-2\)  
b) \(-\frac{10}{3}\)  
c) \(-\frac{1}{3}\)  
d) \(-3\)  
e) 0

\[ 2y y' + y + x \cdot y' = 0 \]

Plug in to get the \( m_{\text{tan}} \)

\[ 2(-2)m_{\text{tan}} + (-2) + (-2)m_{\text{tan}} = 0 \]

\[ -6m_{\text{tan}} - 2 = 0 \]

\[ -6m_{\text{tan}} = 2 \]

\[ m_{\text{tan}} = \frac{2}{-6} = -\frac{1}{3} \]

\[ (2y + x)y' = -y \]

\[ y' = \frac{-y}{2y + x} \]
11. Which of the following statements is true about the curve 
\[(2 + \cos t)i + (1 + \sin t)j.\]

a) Clockwise movement around the circle 
\[(x - 2)^2 + (y - 1)^2 = 1\]

b) Counterclockwise movement around the circle 
\[(x - 2)^2 + (y - 1)^2 = 1\]

\[\text{Clockwise movement around the ellipse} \]
\[x^2/4 + y^2 = 1\]

\[\text{Counterclockwise movement around the ellipse} x^2/4 + y^2 = 1\]

e) None of the above statements is correct.

12. Let \(f(x)\) be a differentiable function and let \(g(x) = 3x^2 - 1.\) Let \(H(x) = f(g(x)),\) the composit of \(f\) and \(g.\) If \(f(0) = 1,\)
\(f'(0) = -1, f(1) = 3, f'(1) = 2, f(2) = -1,\)
\(f'(2) = 5,\) find \(H'(1).\)

a) 30
b) 12
c) -6
d) 6
e) 5
13. What is the domain of \( \ln(x^2 - 4) \)?

a) \( |x| \geq 2 \)

b) \( |x| > 2 \)

c) \( |x| \leq 2 \)

d) \( |x| < 2 \)

e) \( x > 0 \)

14. \( \lim_{{x \to \infty}} 3^{1-x} = \)

a) 0

b) \( \infty \)

c) \( -\infty \)

d) 1

e) 3

\[ \lim_{{x \to -\infty}} 1-x = -\infty \]
15. Find the domain and range of the inverse of \( f(x) = \frac{3x - 5}{7x + 2} \).

- Domain: All real numbers except \( \frac{2}{7} \).
- Range: All real numbers except \( \frac{3}{7} \).

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- Range: All real numbers except \( \frac{3}{7} \).

- Domain: All real numbers except \( \frac{3}{2} \).
- Range: All real numbers except \( \frac{2}{3} \).

- Domain: All real numbers except \( \frac{5}{3} \).
- Range: All real numbers.

- None of the above is correct.

\[
y = \frac{3x - 5}{7x + 2}
\]

\[
7y + 2y = 3x - 5
\]

\[
2y + 5 = 3x - 7xy
\]

\[
2y + 5 = (3 - 7y)x
\]

\[
2y + 5 = x
\]

16. If \( (\cos 3t, t) \) is the position of an object at time \( t \), find the acceleration of the object at time \( t = \frac{\pi}{9} \).

- \( a = \langle -9 \cos (3t), 0 \rangle \)
  
  \[
  a \left( \frac{\pi}{9} \right) = \langle -9 \cos \left( \frac{\pi}{3} \right), 0 \rangle
  = \langle -\frac{9}{2}, 0 \rangle
  \]
17. If \( f(x) = e^{x \tan x} \), find \( f'(x) \).
   
   a) \( f'(x) = e^{x \tan x} \)
   
   b) \( f'(x) = \sec^2 x e^{x \tan x} \)
   
   c) \( f'(x) = (\tan x + x \sec^2 x) e^{x \tan x} \)
   
   d) \( f'(x) = (\tan x + x \sec x \tan x) e^{x \tan x} \)
   
   e) \( f'(x) = x \tan x e^{x \tan x - 1} \)

\[
\frac{d}{dx} = \left(1 + \frac{1}{\tan x} + x \sec^2 x\right) e^{x \tan x}.
\]

18. Find the equation of the tangent line to the graph of \( x = e^{2t} \), \( y = te^t \) at the point \((1,0)\).

   a) \( y = 2x - 1 \)
   
   b) \( y = 4x - 4 \)
   
   c) \( y = \frac{1}{2} - \frac{1}{2} \)
   
   d) \( y = \frac{1}{3} x - \frac{1}{3} \)
   
   e) \( y = x - 1 \)

\[
\frac{dy}{dx}\bigg|_{t=0} = \frac{d\frac{dy}{dt}}{d\frac{dx}{dt}}\bigg|_{t=0} = \frac{e^{2t}}{2e^{2t}} = \frac{1}{2} = m_{tan}
\]

\[
\text{tangent line: } y - 0 = \frac{1}{2}(x - 1) \quad \Rightarrow \quad y = \frac{1}{2}x - \frac{1}{2}
\]
19. Find the quadratic approximation for \( f(x) = \frac{1}{x} \) at \( x = 1 \).

a) \( x^2 - 3x + 3 \) 

\[ Q = f(a) + f'(a)(x-a) + \frac{f''(a)}{2!}(x-a)^2 \]

\[ = 1 + (-1)(x-1) + \frac{2}{2}(x-1)^2 \]

\[ = 1 - x + 1 + \left( x^2 - 2x + 1 \right) \]

\[ = x^2 - 3x + 3 \]

\[ f(x) = x^{-1} \quad a = 1 \]

\[ f'(x) = -x^{-2} = -\frac{1}{x^2} \]

\[ f''(x) = 2x^{-3} = \frac{2}{x^3} \]

b) \( x^2 - x + 2 \)

c) \( x^2 - 2x + 1 \)

d) \( x^2 + 4x + 5 \)

e) \( x^2 + x - 3 \)

20. a.) Find the linear approximation for \( f(x) = \sqrt{x+1} \) at \( x = 0 \).

b.) Use part a.) to obtain an approximation to \( \sqrt{1.01} \)

\[ L = f(0) + f'(0)(x-0) \]

\[ = 1 + \frac{1}{4}x \]

\[ L(x) = 1 + \frac{1}{4}x \]

\[ L(\cdot01) = 1 + \cdot01 \]

\[ = 1.0025 \]
21. The position of a particle is given by \( \mathbf{r}(t) = \left\langle \frac{\cos t}{e^t}, \frac{\sin t}{e^t} \right\rangle. \) 

Find the velocity and speed of the particle when \( t = 0. \)

\[
\mathbf{r}(t) = \left\langle e^{-t} \cos t, e^{-t} \sin t \right\rangle
\]
\[
\mathbf{v}(t) = \left\langle -e^{-t} \cos t - e^{-t} \sin t, -e^{-t} \sin t + e^{-t} \cos t \right\rangle
\]
\[
\mathbf{v}(0) = \left\langle -1, 1 \right\rangle
\]

Speed = \( |\mathbf{v}(0)| = \sqrt{(-1)^2 + 1^2} = \sqrt{2} \)

22. The radius of a sphere was given to be 8 inches with a maximum possible error in measurement of 0.01 inches. Use differentials to estimate the maximum error in the calculated volume of the sphere.

\( r = 8 \quad dr = \Delta r = 0.01 \)

\[
V = \frac{4}{3} \pi r^3 \quad \text{find } dV
\]
\[
dV = 4 \pi r^2 \, dr
\]

\[
dV = 4 \pi (8)^2 (0.01) = 2.56 \pi \text{ cubic in.}
\]
23. Find all values of $x$ between 0 and $2\pi$ where the tangent line to $f(x) = 2x - \tan x$ is horizontal.

\[
f'(x) = 2 - \sec^2 x
\]

\[
0 = 2 - \sec^2 x
\]

\[
\sec^2 x = 2
\]

\[
\frac{1}{\cos^2 x} = 2
\]

\[
1 = 2 \cos^2 x
\]

\[
\frac{1}{2} = \cos^2 x
\]

\[
\cos x = \pm \frac{\sqrt{2}}{2}
\]

\[
\cos x = \frac{\sqrt{2}}{2}
\]

\[
x = \frac{\pi}{4}, \frac{7\pi}{4}
\]

\[
\cos x = -\frac{\sqrt{2}}{2}
\]

\[
x = \frac{3\pi}{4}, \frac{5\pi}{4}
\]

24. A trough is 20 feet long. The end of the trough is an isosceles triangle with height 10 feet and length of 3 feet across the top. If water is poured in the trough at a rate of 3 cubic feet per minute how fast is the water level rising when the height of the water is 1 foot?

\[
\frac{dV}{dt} = 3 \text{ ft}^3/\text{min}
\]

\[
h = 1 \text{ ft} \quad \text{then} \quad \frac{dh}{dt} = ?
\]

\[
V = \frac{1}{2} b \cdot h \cdot L = \frac{1}{2} b \cdot h (20)
\]

\[
V = 10 \cdot b \cdot h
\]

\[
V = 10 \left( \frac{3}{10} h \right) (h)
\]

\[
V = 3h^2
\]

\[
\frac{dV}{dt} = 6h \frac{dh}{dt}
\]

\[
3 = 6 \frac{dh}{dt}
\]

\[
\frac{dh}{dt} = \frac{3}{6} = \frac{1}{2} \text{ ft/min}
\]
25. Starting with \( x_1 = 2 \), apply Newton's Method once to get an approximate solution to \( \sqrt[3]{x^3 - 2x - 5} = 0 \).

\[
\frac{y - f(a)}{f'(a)} = \frac{x - a}{f'(x)}
\]

Plug in \( a \) for \( y \) and solve for \( x \):

\[
x_2 = 2 - \frac{f(2)}{f'(2)} = 2 - \frac{8 - 4 - 5}{12 - 2} = 2 - \frac{1}{10} = 2 + \frac{1}{10} = 2.1
\]

26. Find an equation (in any form) of the line tangent to the curve \( r(t) = (t^6 + t^2)i + (t^4 + t^2)j \) at the point where \( t = 1 \).

\[
r' = \langle 6t^5 + 3t^2, 4t^3 + 2t \rangle
\]

Tangent vector \( r'(1) = \langle 9, 6 \rangle \)

\[
r(1) = \langle 2, 2 \rangle
\]

Vector (parameterized)

\[
\langle 2, 2 \rangle + t \langle 9, 6 \rangle = \langle 2 + 9t, 2 + 6t \rangle
\]

Parameter (Cartesian)

\[
x = 2 + 9t, y = 2 + 6t
\]

Slope of the tangent line

\[
m = \frac{\Delta y}{\Delta x} = \frac{\frac{9}{4}(x-2)}{x-2} = 9
\]

Equation of the tangent line

\[
y - 2 = \frac{9}{4}(x - 2)
\]
27. A rope is attached to the bow of a boat coming in for the evening. Assume the rope is drawn in over a pulley 5 feet higher than the bow at a rate of 2 feet per second. How fast is the boat docking when the length of the rope from the bow to the pulley is 13 feet?

\[ \frac{dx}{dt} = -2 \text{ ft/sec} \]

\[ y^2 + 25 = x^2 \]

\[ 2y \frac{dy}{dt} = 2x \frac{dx}{dt} \]

\[ \frac{dy}{dt} = \frac{2x}{2y} \cdot \frac{dx}{dt} \]

\[ \frac{dy}{dt} = \frac{2(13)}{2(12)} \cdot (-2) = -\frac{13}{6} \text{ ft/sec} \]

Docking at a speed of \( \frac{13}{6} \) ft/sec.