Marginal analysis in Business and Economics

1. The price demand function for a product is given by \( p = 0.05x^3 + 5x \), where \( p \) is in dollars and \( x \) is in items.
   (a) Find the marginal revenue function.
   (b) Compute \( R'(5) \) and interpret the value.
   (c) Use the marginal revenue function to estimate the revenue of the
      i. 8th item sold.
      ii. 15th item sold.
   (d) Compute the exact revenue for the
      i. 8th item sold.
      ii. 15th item sold.

2. Susan sells used CDs on an on-line store. Her total profit (in dollars) from the sale of \( x \) CDs is given by \( P(x) = 11x - 0.05x^2 - 200 \).
   (a) Find the marginal profit function.
   (b) Compute \( P'(25) \) and interpret the value.
   (c) Use the marginal profit function to estimate the profit of the
      i. 60th CD sold.
      ii. 84th CD sold.
   (d) Compute the exact profit for the
      i. 60th CD sold.
      ii. 84th CD sold.

3. A business has determined cost model for one of its products.
   \( C(x) = 5x^2 - 3x + 2500 \)
   where \( x \) has units of thousand of items produced and \( C(x) \) has units of dollars.
   (a) Compute \( C'(8) \) and interpret this result.
   (b) Compute \( C'(15) \) and interpret this result.
   (c) Compute \( C'(15) \)
   (d) Find the average cost when 20,000 items are produced.
   (e) Find the marginal average cost when 20,000 items are produced.
   (f) Compute the average rate of change of cost when the number of items produced and sold goes from 5,000 items to 15,000 items.
   (g) Compute the average rate of change of the cost when the number of items produced goes from 10,000 to 30,000.

4. A business has determined cost and revenue models for one of its products.
   \( C(x) = 11x^2 + 7500 \)
   \( R(x) = x^3 - 6x^2 + 65x \)
   \( x \) is the number of items that are produced and sold. \( C(x) \) and \( R(x) \) both have units of dollars.
   (a) Find the marginal average profit function.
   (b) Find the average profit when 38 items are produced.
   (c) Find the marginal average profit when 38 items are produced.
   (d) Use the results from (4b) and (4c) to estimate the average profit per item if 39 items are sold.
   (e) Use the results from (4b) and (4c) to estimate the average profit per item if 40 items are sold.
   (f) Compute \( \overline{R}(40) \) and compare it to the result in (4e).

5. The revenue function for a product is given by \( R(x) = 0.05x^4 + 5x^2 \), where \( R(x) \) is in dollars and \( x \) is in items.
   (a) Find the marginal average revenue function.
   (b) Compute \( \overline{R}(5) \) and interpret the value.
   (c) Compute \( \overline{R}(5) \)
   (d) Use the results from (5b) and (5c) to estimate the average revenue per item if 6 items are sold.
   (e) Use the results from (5b) and (5c) to estimate the average revenue per item if 8 items are sold.
Answers

1. (a) \( R'(x) = 0.2x^3 + 10x \)
(b) \( R'(5) = 75 \)
   When 5 items are sold, the revenue is increasing at a rate of 75 dollars per item. This means that the approximate revenue of the next item, 6th item, will be $75.
(c) i. \( R'(7) = 138.60 \)
   revenue of the 8th item is approximately $138.60
ii. \( R'(14) = 688.80 \)
   revenue of the 15th item is approximately $688.80
(d) i. \( R(8) - R(7) = 159.75 \)
   ii. \( R(15) - R(14) = 755.45 \)

2. (a) \( P'(x) = -0.1x + 11 \)
(b) \( P'(25) = 8.5 \)
   When 25 CDs are sold, the profit is increasing at a rate of $8.50 per CD. This means that the approximate profit for the 26th(next) CD is $8.50.
(c) i. \( P'(59) = 5.1 \)
   profit of the 60th CD is approximately $5.10
ii. \( P'(83) = 2.7 \)
   profit of the 84th CD is approximately $2.70
(d) i. \( P(60) - P(59) = 5.05 \)
   ii. \( P(84) - P(83) = 2.65 \)

3. \( C(x) = \frac{5x^2 - 3x + 2500}{x} = 5x - 3 + 2500x^{-1} \)
   and \( C'(x) = 5 - 2500x^{-2} = \frac{5x^2 - 2500}{x^2} \)
(a) \( C(8) = 349.5 \)
   The average cost to produce each item when 8,000 items are produced is $349.50.
(b) \( C(15) = 238.67 \)
   The average cost to produce each item when 15,000 items are produced is $238.67.
(c) \( C'(15) = -6.11 \)
(d) $222.00
(e) \( C'(20) = -1.25 \)
(f) \( \frac{C(15) - C(5)}{15 - 5} = 97 \)
(g) \( \frac{C(30) - C(10)}{30 - 10} = 197 \)

4. \( \frac{P}{x} = \frac{x^3 - 17x^2 + 65x - 7500}{x} \)
   or \( \frac{P}{x} = x^2 - 17x + 65 - 7500x^{-1} \)
(a) \( P'(x) = 2x - 17 + 7500x^{-2} = \frac{2x^3 - 17x^2 + 7500}{x^2} \)
(b) \( P(38) = 665.63 \)
(c) \( P'(38) = 64.19 \)
(d) $665.63 + 64.19 = $729.82
(e) $665.63 + (2)64.19 = $794.01
(f) \( \frac{P(40)}{x} = 797.50 \)

5. (a) \( R'(x) = 0.15x^2 + 5 \)
(b) \( R(5) = 31.25 \)
   The average revenue per item is $31.25 when 5 items are being sold.
(c) \( R'(5) = 8.75 \)
(d) \( 31.25 + 8.75 = $40 \)
(e) \( 31.25 + 8.75(3) = $57.50 \)