Applications in Business and Economics

1. The rate of change of the income produced by an investment is given by \( f(x) = 4000e^{0.04x} \)
   
   (a) What is the total income produced during the first 5 years.
   
   (b) Find the future value of this income stream at 8%, compounded continuously.
   
   (c) Find the present value of this income stream. This is amount that if deposited into an account now at 8% will produce the same future value found in part b.
   
   (d) If this investment was being offered for sale, what would be the largest price that you would pay for this investment?

2. Find the present value of a continuous income stream with rate of flow \( f(x) = 3500e^{0.12x} \) when \( T = 10 \) and \( r = 7\% \).

3. An oil well is being offered for sale for $310,000 and has a rate of change of income per year given by \( f(x) = 100e^{-0.1x} \), in thousands of dollars, and a useful life of 5 years. Would you buy if the current interest rate \( r \) that can be compounded continuously over the next 5 years is
   
   (a) 10\%
   
   (b) 12\%

4. Find the consumers’ surplus for the demand function and the given demand level.
   
   (a) \( p = D(x) = 2720 - 0.04x^2 \), \( x = 100 \)
   
   (b) \( p = D(x) = 320e^{-0.02x} \), \( x = 175 \)

5. Find the consumers’ surplus for the demand function \( p = D(x) = 1610 - 0.08x^2 \) when price is $810.

6. Find the producers’ surplus for the supply function and the given demand level.
   
   (a) \( p = S(x) = 25e^{0.03x} \), \( x = 55 \)
   
   (b) \( p = S(x) = 0.01x^2 + 25 \), \( x = 190 \)

7. Find the producers’ surplus for the supply function \( p = S(x) = 0.05x^2 + 20 \) when the price is $461.8

Answers

1. (a) Total income = \( \int_{0}^{5} 4000e^{0.04x} \, dx = 22,140.28 \)
   
   (b) \( FV = e^{0.08\times 5} \int_{0}^{5} 4000e^{0.04x} e^{-0.08x} \, dx = 27,042.19 \)
   
   (c) \( PV = \int_{0}^{5} 4000e^{0.04x} e^{-0.08x} \, dx = 18,126.92 \)
   
   (d) Largest price would be $18,126.92 since this is the amount that, if invested at 8%, will equal the future value of the investment in 5 years.

2. \( PV = \int_{0}^{10} 3500e^{0.12x} e^{-0.07x} \, dx = 45,410.49 \)

3. (a) \( PV = \int_{0}^{5} 100e^{-0.1x} e^{-0.1x} \, dx = 316.06027 \) thousand dollars
   
   yes since the present value of the well is $316,060 and the price of the well is less that this amount.
   
   (b) \( PV = \int_{0}^{5} 100e^{-0.1x} e^{-0.12x} \, dx = 303.2404 \) thousand dollars
   
   no since the present value of the well is $303,240 and the price of the well is more than this amount.

4. (a) first find \( p, p = 2720 - 0.04(100)^2 = 2320 \)
   
   \( CS = \int_{0}^{100} [(2720 - 0.04x^2) - 2320] \, dx = 26666.67 \)
   
   (b) \( CS = \int_{0}^{175} [320e^{-0.02x} - 9.66] \, dx = 13,826.34 \)

5. \( CS = \int_{0}^{100} [(1610 - 0.08x^2) - 810] \, dx = 53,333.33 \)

6. (a) \( PS = \int_{0}^{55} (130.17 - 25e^{0.03x}) \, dx = 3,653.53 \)
   
   (b) \( PS = \int_{0}^{190} (386 - [0.01x^2 + 25]) \, dx = 45,726.67 \)

7. \( PS = \int_{0}^{94} (461.8 - [0.05x^2 + 20]) \, dx = 27,686.13 \)