The Fundamental Theorem of Calculus

Evaluate the integrals in problems 1-5. The letter A is some positive number greater than 5.

1. \( \int_{1}^{A} \left( x\sqrt{x} + \frac{7}{x^2} + 3 \right) \, dx \)

2. \( \int_{2}^{A} -2e^{(4-2x)} \, dx \)

3. \( \int_{0}^{A} \frac{30x^4 - 16}{3x^5 - 8x + 16} \, dx \)

4. \( \int_{1}^{A} (4x - 3x^2)(22x^2 - 11x^3)^6 \, dx = \)

5. \( \int_{A}^{A} (4x - 3x^2)(22x^2 - 11x^3)^6 \, dx = \)

6. Suppose the rate of sales of an item is given by \( S'(x) = -3x^2 + 36x \), where \( x \) is the number of weeks after an advertising campaign has begun.
   
   (a) How many items were sold during the first two weeks?
   
   (b) How many items were sold during the fourth week?

7. A firm estimates the rate that the revenue generated by a new product can be modeled by \( R'(x) = 700xe^{-0.5x^2} \), where \( R'(x) \) is in thousand of dollars per year and \( x \) is in years.
   
   (a) Find the revenue for the product during the second year.
   
   (b) Find the revenue for the product for the first three years.

8. From the time a new product was introduced, sales in thousands of the product have been increasing at a rate of \( S'(x) = 2 + .5x \), where \( x \) is the time since the introduction of the new product. At the beginning of the third year, an advertising campaign was introduced and sales then increased at the rate \( S'(x) = 3x^2 - 5x + 1 \). Find the total sales during the first four years.

9. Water is being pumped into a swimming pool at a rate \( r(x) \), in gallons per hour, and \( x \) is in hours. Interpret the following with respect to the swimming pool. Be sure to use units with your answer.

\[ \int_{0}^{5} r(x) \, dx = 3,520 \]
Answers

1. \[ \int_{1}^{A} \left( x^{1.5} + 7x^{-2} + 3 \right) dx = \left. \frac{x^{2.5}}{2.5} - 7x^{-1} + 3x \right|_{1}^{A} = \left( \frac{A^{2.5}}{2.5} - \frac{7}{A} + 3A \right) - \left( \frac{1^{2.5}}{2.5} - \frac{7}{1} + 3 \right) = 0.4A^{2.5} - \frac{7}{A} + 3A + 3.6 \]

2. \[ u = 4 - 2x \]
\[ \frac{du}{dx} = -2 \]
\[ du = -2dx \]
\[ \int_{2}^{A} -2e^{(4-2x)} dx = \int_{2}^{A} e^{u} du = \left. e^{u} \right|_{x=2}^{x=A} = e^{A} - e^{2} = e^{A-2} - 1 \]

3. \[ u = 3x^5 - 8x + 16 \]
\[ \frac{du}{dx} = 15x^4 - 8 \]
\[ du = (15x^4 - 8) dx \]
\[ \int_{0}^{A} \frac{2(15x^4 - 8)}{3x^5 - 8x + 16} dx = \int_{x=0}^{x=A} 2 \ln(u) du = 2 \ln(3x^5 - 8x + 16) \bigg|_{0}^{A} = 2 \ln(3A^5 - 8A + 16) - 2 \ln(16) \]

4. \[ u = 22x^2 - 11x^3 \]
\[ \frac{du}{dx} = 44x - 33x^2 \]
\[ du = (44x - 33x^2) dx \]
\[ \frac{1}{11} du = (4x - 3x^2) dx \]
\[ \int_{1}^{A} (4x - 3x^2)(22x^2 - 11x^3)^6 dx = \int_{x=1}^{x=A} \frac{1}{11}u^6 du = \left. \frac{1}{11}(u^7) \right|_{x=1}^{x=A} = \frac{1}{11}(22A^2 - 11A^3)^7 - \frac{1771561}{7} \]

5. 0 since the limits of integration start at \( x = A \) and also end at \( x = A \).

6. (a) \[ \int_{0}^{2} (-3x^2 + 36x) dx = 64 \text{ items} \]
(b) \[ \int_{3}^{4} (-3x^2 + 36x) dx = 89 \text{ items} \]

7. (a) \[ \int_{1}^{2} 700xe^{-0.5x^2} dx = 329.826 \]
Answer: $329,826

(b) \[ \int_{0}^{3} 700xe^{-0.5x^2} dx = 692.224 \]
Answer: $692,224

8. \[ \int_{0}^{2} (2 + 0.5x) dx + \int_{2}^{4} (3x^2 - 5x + 1) dx = 33 \text{ thousand products} \]

9. 3,520 gallons of water are pumped into the pool during the first 5 hours.

Note: We can not say that this is the total amount of water in the pool since we were not told if the pool was empty at the start.