Exponential Functions

In problems 1-13, solve for x.

1. \(3^x = \frac{1}{81}\)
2. \(7^{2x} = 7\)
3. \(5^{2x-7} = 1\)
4. \(3^{x^2-3} = 3\)
5. \(2^x 4^8 = 8^3\)
6. \(3^x 9^{x+1} = 27\)
7. \(4^{(5x^2-x)} = \frac{1}{16^x}\)
8. \(3^{(2x^2-3x)} = 9^x\)
9. \(x^2 5^{2x} = 5^{2x}\)
10. \((x^2 - 6x + 8)3^x = 0\)
11. \(x^2 6^x = x 6^x\)
12. \(\frac{8}{2^x} = 4^x\)
13. \(\frac{4^{2x}}{2^{x-1}} = 8^{x^2}\)

14. A town grows continuously and has a growth rate of 12%, if 400 people live in the town now, what will be the population in five years? Round your answer to the nearest person.

15. A town grows continuously and has a growth rate of 5%. You are told that in 10 years there will be 10,400 people in the town. How many people are living in the town now? Round your answer to the nearest person.

16. You start an account with $1000. The account pays 6.75% interest compounded monthly. At the end of the first year you deposit an additional $300 and at the end of the second year you deposit an additional $600. What is the balance of the account 6 years after it was started?

17. An account is started with $2000 and it grows to $5000 in 7 years. Find the interest rate the account earned if it the interest was compounded
   (a) annually.
   (b) quarterly.
   (c) monthly.

18. The table gives the number of people (in millions) with cell phones for these years.\(^1\)

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<td>0.5</td>
<td>0.8</td>
<td>1.4</td>
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<tbody>
<tr>
<td>Number</td>
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<td>5.7</td>
<td>8</td>
<td>11</td>
<td>13.8</td>
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Use exponential regression to find the best fitting exponential function. Let \(x = 0\) correspond to 1984. Use the model to estimate the number of people with cell phones in 1999.

19. The consumption of bottled water in the United States has increased dramatically in the past 20 years. The table shows the consumption (gallons per person per year) for the indicated years.\(^2\)

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<td>Consumption</td>
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<td>4.5</td>
<td>8</td>
<td>12.2</td>
<td>17.4</td>
<td>22.0</td>
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</table>

Use exponential regression to find the best fitting exponential function. Let \(x = 0\) correspond to 1980. Use the model to estimate the consumption in 2005.

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\(^1\) Calculus Applications and Technology for Business, Social and Life Sciences, 2nd ed. by Tomastik. page 68

\(^2\) Calculus Concepts, 4th ed. by LaTorre, Kenelly, Reed, Carpenter, Harris, and Biggers. pg. 55
1. $3^x = 3^{-4}$
   Answer: $x = -4$

2. $2^{2x} = 2^1$
   $2x = 1$
   Answer: $x = 0.5$

3. $5^{2x-7} = 5^0$
   $2x - 7 = 0$
   Answer: $x = 3.5$

4. $x = 2$ or $x = -2$

5. $2^x(2^2)^8 = (2^3)^3$
   $2^x = 2^9$
   $2^x = 2^9$
   $x = 9$
   Answer: $x = -7$

6. $3^x + 3^{2x+2} = 3^3$
   $3^{3x+2} = 3^3$
   $3x + 2 = 3$
   Answer: $x = 1/3$

7. $4^{(5x^2-x)} = 4^{-2x}$
   $5x^2 - x = -2x$
   $5x^2 + x = 0$
   $x(5x + 1) = 0$
   Answer: $x = 0$, $x = -0.2$

8. $3^{(2x^2-3x)} = 3^{2x}$
   $2x^2 - 3x = 2x$  
   $x(2x - 5) = 0$
   Answer: $x = 0$, $x = 2.5$

9. $2^{5x^2 - 5^{2x}} = 0$
   $(x^2 - 1)5^{2x} = 0$
   $x^2 - 1 = 0$ since $5^{2x} \neq 0$
   Answer: $x = 1$, $x = -1$

10. $x = 4$, $x = 2$

11. $x = 0$, $x = 1$

12. $2^3 + 2^{-x^2} = 2^{2x}$
   $2^{3-x^2} = 2^{2x}$
   $3 - x^2 = 2x$
   $0 = (x + 3)(x - 1)$
   Answer: $x = -3$, $x = 1$

13. $\frac{(2^2)^{3x}}{2^{2x-1}} = 2^{3x^2}$
   $2^{4x} \cdot 2^{-2(2x-1)} = 2^{3x^2}$
   $2^{2x+1} = 2^{4x^2}$
   $2x + 1 = 3x^2$
   $0 = (3x + 1)(x - 1)$
   Answer: $x = -\frac{1}{3}$, $x = 1$

14. $y = 400e^{0.12x}$
   Answer: 729 people

15. $y = ae^{0.05x}$ where $a$ is the starting amount.
   solve $10400 = ae^{0.05 \times 10}$ for $a$
   Answer: $a = 6308$

16. balance at end of year 1:
   $1000 \left(1 + \frac{0.0675}{12}\right)^{12+1} + 300 = 1369.63$
   balance at end of year 2:
   $1369.63 \left(1 + \frac{0.0675}{12}\right)^{12+2} + 600 = 2064.99$
   balance at the end of 6 years:
   $2064.99 \left(1 + \frac{0.0675}{12}\right)^{12 \times 6} = 2703.02$

17. (a) solve $5000 = 2000 \left(1 + \frac{r}{1}\right)^{(1 \times 7)}$
   
   $2.5 = (1 + r)^7$
   $(2.5)^{1/7} = 1 + r$
   $r = .13985228$
   Answer: 13.9852%

(b) solve $5000 = 2000 \left(1 + \frac{r}{4}\right)^{(4 \times 7)}$

   $2.5 = \left(1 + \frac{r}{4}\right)^{28}$
   $(2.5)^{1/28} = 1 + \frac{r}{4}$
   $4 * ((2.5)^{1/28} - 1) = r$
   Answer: 13.3064%

(c) solve $5000 = 2000 \left(1 + \frac{r}{12}\right)^{(12 \times 7)}$

   Answer: 13.1615%

18. $y = 0.298908(1.5861)^x$
   number of people with cell phones in 1999 is about 302.34028 million people.

19. $y = 2.71584(1.099333)^x$
   bottled water consumption in 2005 is 28.98 gallons per person per year.