Curve Sketching Techniques

For each problem, sketch a graph of a function that has all of the listed properties.

1. Continuous for all real numbers.
   Differentiable for all real numbers.
   \( f''(-1) = 0, f''(1) = 0 \)
   \( f(-1) = 4, f(1) = 0 \)
   \( f'(x) < 0 \) on \((-1, 1)\).
   \( f'(x) > 0 \) on \((-\infty, -1)\) and \((1, \infty)\).
   \( f''(x) < 0 \) on \((-\infty, 0)\).
   \( f''(x) > 0 \) on \((0, \infty)\).

2. Continuous for all real numbers.
   Differentiable for all real numbers.
   x-intercepts 0, 4, and -4.
   \( f'(2) = 0, f'(-2) = 0, f''(0) = 0 \)

3. Continuous for all real numbers except \( x = 3 \)
   Differentiable for all real numbers except \( x = 3 \)
   Critical value at \( x = 5 \)
   \( \lim_{x \to \infty} f(x) = 0, \lim_{x \to -\infty} f(x) = 0 \)

4. Continuous for all real numbers except \( x = -2, 0, 2 \)
   Differentiable for all real numbers except \( x = -2, 0, 2 \)
   Inflection points at \((-1, 0)\) and \((1, 0)\).
   Vertical Asymptote: \( x = -2, x = 2, \) and \( x = 0 \).
   \( \lim_{x \to \infty} f(x) = 0 \) and \( \lim_{x \to -\infty} f(x) = 0 \)
   \( f'(x) < 0 \) on \((-\infty, -2)\) and \((-2, 0)\).
   \( f'(x) > 0 \) on \((0, 2)\) and \((2, \infty)\).
   \( f''(x) > 0 \) on \((-\infty, -1)\) and \((1, 2)\).
   \( f''(x) < 0 \) on \((-\infty, -2), (-1, 0), (0, 1), \) and \((2, \infty)\).

5. Domain: all real numbers except \( x = 2 \) and \( x = -2 \)
   Continuous for all real numbers except \( x = -2, 2 \)
   Not differentiable at \( x = -2, 2 \)
   x-intercept: 0
   y-intercept: 0
   Vertical asymptote: \( x = -2 \) and \( x = 2 \)
   Horizontal asymptote: none
   Relative extreme at the point \((4, -4)\)
   Relative minimum at the points \((-4, 4)\)
   Inflection point: \((0, 0)\)
   \( f'(x) > 0 \) on \((-4, -2), (-2, 2), \) and \((2, 4)\)
   \( f'(x) < 0 \) on \((-\infty, -4), \) and \((4, \infty)\)
   \( f''(x) > 0 \) on \((-\infty, -2)\) and \((0, 2)\)
   \( f''(x) < 0 \) on \((-2, 0), \) and \((2, \infty)\)

6. Continuous and differentiable for all real numbers.
   \( f'(-1) = 0 \) and \( f''(5) = 0 \)
   \( f'(x) > 0 \) on \((-1, 5)\) and \((5, \infty)\)
   \( f'(x) < 0 \) on \((-\infty, -1)\)
   \( f''(x) > 0 \) on \((-\infty, 2)\) and \((5, \infty)\)
   \( f''(x) < 0 \) on \((2, 5)\)

7. Continuous for all real numbers except \( x = 1 \) where it has a vertical asymptote.
   Differentiable everywhere except at \( x = 1 \) and \( x = 5 \)
   Horizontal asymptote of \( y = 0 \).
   \( f'(5) = \text{DNE} \) and \( f(5) = 4 \)
   \( f'(x) < 0 \) on \((5, \infty)\)
   \( f'(x) > 0 \) on \((-\infty, 1)\) and \((1, 5)\)
   \( f''(x) < 0 \) on \((1, 5)\)
   \( f''(x) > 0 \) on \((-\infty, 1)\) and \((5, \infty)\)

8. Continuous for all real numbers.
   Differentiable everywhere except at \( x = 0 \)
   Horizontal asymptote of \( y = 5 \).
   \( f'(2) = 0 \) and \( f(2) = 1 \)
   \( f'(x) < 0 \) on \((-\infty, 0)\)
   \( f'(x) > 0 \) on \((0, 2)\) and \((2, \infty)\)
   \( f''(x) < 0 \) on \((-\infty, 0)\) and \((0, 2)\) and \((4, \infty)\)
   \( f''(x) > 0 \) on \((2, 4)\)

9. Continuous for all real numbers.
   Differentiable everywhere except at \( x = 2 \)
   Horizontal asymptote of \( y = 3 \).
   \( f'(6) = 0 \) and \( f(6) = 6 \)
   \( f''(8) = 0 \)
   \( f'(x) < 0 \) on \((-\infty, 2)\) and \((6, \infty)\)
   \( f'(x) > 0 \) on \((2, 6)\)
   \( f''(x) < 0 \) on \((2, 8)\)
   \( f''(x) > 0 \) on \((-\infty, 2)\) and \((8, \infty)\)
Answers

1. Graph of $f(x)$.

2. Graph of $f(x)$.

3. Graph of $f(x)$.

4. Graph of $f(x)$.

5. Graph of $f(x)$.

6. Graph of $f(x)$.

7. Graph of $f(x)$.

8. Graph of $f(x)$.

9. Graph of $f(x)$.