Basic Differentiation Properties

In problems 1-4 compute \( y' \)

1. \( y = x^2(x^3 + 3x + 7) \)
2. \( y = (x^3 + 4x + 1)\sqrt{x} \)
3. \( y = \sqrt[3]{x^3} + \sqrt[3]{x^2} + 7^2 \)
4. \( y = \frac{14}{\sqrt[3]{x^{10}}} + \pi^4 + x^{1.8} \)

5. Find where the function \( f(x) = x^3 - 5x^2 + 6x - 30 \) has an instantaneous rate of change of 6.
6. Find the values of \( x \) where the tangent line for the function \( y = (x^2 + 6)(x + 5) \) has a slope of 14
7. Find where the function \( f(x) = x^3 - 6x^2 - 56x + 25 \) has an instantaneous rate of change of 40.
8. Find the value of \( B \) so that \( f(x) = x^3 + Bx^2 + 4 \) will have instantaneous rate of change of 30 at \( x = 2 \).
9. Find the value of \( B \) so that \( f(x) = x^4 - 3Bx^2 + 7x + 2 \) so that \( f'(3) = -29 \).
10. Find the value of \( x \) where the tangent line at \( x = 3 \) to the function \( y = x^2 + 3 \) will cross the x-axis.
11. Find the value of \( x \) where the tangent line at \( x = 4 \) to the function \( y = x^2 + 2x + 1 \) will cross the x-axis.

Answers

1. \( y = x^5 + 3x^3 + 7x^2 \)
   \( y' = 5x^4 + 9x^2 + 14x \)

2. \( y = x^{3.5} + 4x^{1.5} + x^{0.5} \)
   \( y' = 3.5x^{2.5} + 6x^{0.5} + 0.5x^{-0.5} \)

3. \( y = x^{3/5} + x^{2/3} + 7^2 \)
   \( y' = \frac{3}{5}x^{-2/5} + \frac{2}{3}x^{-1/3} \)

4. \( y = 14x^{-10/7} + \pi^4 + x^{1.8} \)
   \( y' = -20x^{-17/7} + 1.8x^{0.8} \)

5. \( y' = 3x^2 - 10x + 6 \)
   \( 6 = 3x^2 - 10x + 6 \)
   \( 0 = 3x^2 - 10x \)
   \( 0 = x(3x - 10) \)

Answer: \( x = 0, x = \frac{10}{3} \)

6. \( x = -4, x = \frac{2}{3} \)

7. \( x = 8, x = -4 \)

8. \( f'(x) = 3x^2 + 2Bx \) and we want \( f'(2) = 30 \)
   \( 30 = 3(2)^2 + 2B(2) \)
   \( 30 = 12 + 4B \)

Answer: \( B = 4.5 \)

9. \( B = 8 \)

10. The equation of the tangent line at \( x = 3 \) is \( y - 12 = 6(x - 3) \) or \( y = 6x - 6 \)
    now set \( y \) to zero and solve for \( x \).

Answer: \( x = 1 \)

11. \( x = 1.5 \)