Section 8.4: The Binomial Distribution

Example: An unfair coin, P(H) = 0.6, is to be flipped 7 times.

A) Find the probability that only the first four flips are heads.

\[ (.6)^4 \cdot (.4)^3 \]

B) Find the probability that only the 1st, 2nd, 5th and last toss are heads.

\[ (.6)^4 \cdot (.4)^3 \]

C) Find the probability that exactly four of the flips are heads.

\[ \binom{7}{4} (.6)^4 (.4)^3 \]

D) Find the probability that only 6 of the flips were heads.

\[ \binom{7}{6} (.6)^6 (.4)^1 \]

Definition: An experiment is called a Bernoulli experiment (Binomial trial/Bernoulli trial) if it has the following conditions.

1) \text{Fixed # of trials (n times)}

2) \text{Independence between trials.}

3) \text{2 possible results (success & failure)}

4) \text{Prob of success stays fixed}

\[ n = \# \text{ of trials} \]
\[ p = \text{prob of success} \quad \Rightarrow \quad p + q = 1 \]
\[ q = \text{prob of failure} \]
\[ r = \# \text{ of successes} \]

\[ \text{P}(X = r) = \binom{n}{r} p^r q^{n-r} \]
Example: A task has a 65% chance of a successful completion. Find the probability of the following number of successes when this task is repeated 70 times. Assume that repetition of this task is independent.

A) exactly 30 failures.

\[
\begin{align*}
N &= 70 \\
\rho &= 0.65 \\
\gamma &= 35 \\
\gamma' &= 40 \\
\binom{70}{40} \cdot (0.65)^{40} \cdot (0.35)^{30} \\
\text{pdf}(70, 0.65, 40) &= 0.0382 \\
\binom{70}{40} &= 1062
\end{align*}
\]

B) at most 40 successes.

\[
\begin{align*}
\gamma &= 0, 1, 2, 3, \ldots, 40 \\
\text{cdf}(70, 0.65, 40) &= 0.1062
\end{align*}
\]

C) at least 50 successes.

\[
1 - \text{cdf}(70, 0.65, 49)
\]

D) between 38 and 48 successes.

\[
bpdf(70, 0.65, 47) - \text{binomcdf}(70, 0.65, 38)
\]

\[
= 0.6469
\]
Calculator Commands: The commands for the binomial distribution are found under the distribution menu, press 2nd VARS.

\[ P(x = r) \] \quad \text{binompdf}(n, p, r) \rightarrow \text{exactly} \ r \ \text{successes.} \]

\[ P(x \leq r) \] \quad \text{binomcdf}(n, p, r) \rightarrow \text{at most} \ r \ \text{successes.} \]

Example: Roll a die until you get a 5. What is the probability that it took 4 rolls?

\[
\left( \frac{5}{6} \right)^3 \left( \frac{1}{6} \right)
\]
Example: In a group of 30 randomly selected people, find the probability that exactly 2 of the people were born in the month of July. Assume that months are equally likely.

\[ \begin{align*} 
\lambda &= 30 \\
\rho &= \frac{1}{12} \\
\tau &= 2 \\
\end{align*} \]

\[ \binom{30}{2} \left( \frac{1}{12} \right)^2 \left( \frac{11}{12} \right)^{28} \]

\[ \text{pdf} \left( \frac{30}{12}, 2 \right) \]

Example: You are taking a 20 question multiple choice exam with each question having 3 answers. Find the probability that you pass the exam if you know 11 of the questions and randomly guess for the rest.

\[ \begin{align*} 
\lambda &= 11 \\
\rho &= \frac{1}{3} \\
\tau &= 3, 4, 5, 6, \ldots, 9 \\
\end{align*} \]

\[ \text{cdf} \left( 9, \frac{1}{3}, 9 \right) - \text{cdf} \left( 9, \frac{1}{3}, 2 \right) \]

\[ .6228 \]
Example: A multiple choice exam has 10 questions where each question has 4 answers. If a student guesses at all of the questions, what is the expected number of questions that the student will get correct? What is the expected grade on the exam?

\[ n = 10 \quad p = \frac{1}{4} \]

\[ n \cdot p = 10 \cdot \frac{1}{4} = 2.5 \]

**Expected grade** = \[10p_{2.5} = 25\]

**Definition:** The expected value and the standard deviation of a binomial distribution with \( n \) trials and probability of success \( p \) on a single trial is

\[ E(X) = n \cdot p \]

\[ \sigma = \sqrt{n \cdot p \cdot q} \]

\[ p + q = 1 \]

Example: The probability of an adverse reaction to a flu shot is 0.17. Flu shots are given to a group of 90 people. Let \( X \) represent the number of people with an adverse reaction. Find the probability that the number of people with an adverse reaction is within 2 standard deviations of the mean.

\[ E(X) = n \cdot p = 90 \cdot (0.17) = 15.3 \]

\[ \sigma = \sqrt{n \cdot p \cdot q} = \sqrt{90 \cdot (0.17) \cdot (0.83)} \]

\[ = 3.5636 \]

\[ \mu - 2\sigma = 15.3 + 2 \cdot 3.5636 = 22.4272 \]

\[ \mu + 2\sigma = 8.1728 \]

\[ \gamma = 9, 10, \ldots, 22 \]

\[ bcdf(90, 0.17, 22) - bcdf(90, 0.17, 8) \]
9, 10, 11, ..., 14
d
120, 21, ..., 30