Section 7.2: Definition of Probability

Definition: Probability is a number that is assigned to an outcome of a sample space that indicates how likely that outcome is to happen when conducting an experiment.

\[ P(r) > P(y) > P(z) \]

Rules of Probability

1) all prob is between 0 and 1 (inclusive)

2) all prob adds up to 1.

<table>
<thead>
<tr>
<th>Die Roll</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Method A</td>
<td>1/6</td>
<td>1/6</td>
<td>1/6</td>
<td>1/6</td>
<td>1/6</td>
<td>1/6</td>
</tr>
<tr>
<td>Method B</td>
<td>0.1</td>
<td>0.2</td>
<td>0.3</td>
<td>0.2</td>
<td>0.1</td>
<td>0.3</td>
</tr>
<tr>
<td>Method C</td>
<td>0.1</td>
<td>0.2</td>
<td>0.3</td>
<td>0.2</td>
<td>0.1</td>
<td>0.1</td>
</tr>
</tbody>
</table>

Definition: A sample space where every outcome has the same probability (chance of happening) is called a uniform sample space and

\[ P(\text{any individual outcome}) = \frac{1}{n(s)} \]

Definition: A probability distribution is a chart that shows the probability of every outcome in the sample space.

Example: A group of people were asked what was their favorite soft drink. The results of the survey are given in the table.

<table>
<thead>
<tr>
<th>drinks</th>
<th>Dr Pepper</th>
<th>Coke</th>
<th>Pepsi</th>
<th>Root beer</th>
<th>other</th>
</tr>
</thead>
<tbody>
<tr>
<td>number of people</td>
<td>175</td>
<td>10</td>
<td>40</td>
<td>60</td>
<td>25</td>
</tr>
</tbody>
</table>

Give the probability distribution that is associated with this survey.

<table>
<thead>
<tr>
<th>drink</th>
<th>prob.</th>
<th>freq.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dr Pepper</td>
<td>175/310</td>
<td>310 people</td>
</tr>
<tr>
<td>Coke</td>
<td>10/310</td>
<td>310 people</td>
</tr>
<tr>
<td>Pepsi</td>
<td>40/310</td>
<td>310 people</td>
</tr>
<tr>
<td>Root beer</td>
<td>60/310</td>
<td>310 people</td>
</tr>
<tr>
<td>other</td>
<td>25/310</td>
<td>310 people</td>
</tr>
</tbody>
</table>
Example: The number of grades for a group of Math 166 students are shown in the table.

<table>
<thead>
<tr>
<th>grade</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>F</th>
</tr>
</thead>
<tbody>
<tr>
<td>frequency</td>
<td>40</td>
<td>84</td>
<td>71</td>
<td>38</td>
<td>31</td>
</tr>
</tbody>
</table>

If a student from this group is selected at random, what is the probability that the student made

A) an A?

\[
\frac{40}{264}
\]

B) an A or a B?

\[
\frac{40 + 84}{264}
\]

Example: You flip a coin 10, 50, and then 100 times. For each of these, how many times will you get a head?

Definition: If \( E \) is an event of a sample space, then

\[
P(E) = \text{sum of the prob of the outcomes in } E.
\]

\[
P(S) = 1 \quad \text{and} \quad P(\emptyset) = 0
\]

if \( S \) is uniform.

\[
P(E) = n(E) \cdot \left( \frac{1}{n(S)} \right) = \frac{n(E)}{n(S)}
\]
Example: A single card is drawn from a standard deck of cards. Find these probabilities.

A) \( P(\text{king}) = \frac{4}{52} \)

B) \( P(\text{heart}) = \frac{13}{52} \)

C) \( P(\text{heart or a king}) = \frac{13 + 4 - 1}{52} = \frac{16}{52} \)

\[ \text{P(A} \cup \text{B)} = \text{P(A)} + \text{P(B)} - \text{P(A} \cap \text{B)} \]

\( n(\text{A} \cup \text{B}) = n(\text{A}) + n(\text{B}) - n(\text{A} \cap \text{B}) \)

D) \( P(\text{not a king or not a queen)} = \)

\[ P(\text{K} \cup \text{Q}^c) = P(\text{K}) + P(\text{Q}^c) - P(\text{K} \cap \text{Q}^c) \]

\[ = \frac{16}{52} + \frac{48}{52} - \frac{4}{52} \]

\[ = 1 \]

\[ \text{K} \quad \text{Q} \quad \text{K}^c \]

\[ \text{Q}^c \]

\[ \text{Union = everything} \]

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Example: Roll two six sided dice (one red and one green)

A) Give the probability distribution for the sum of the die.

B) Find the probability that the sum of the die is a multiple of 5.
Example: Roll a 6 sided die. Suppose that any even number is twice as likely to happen as any odd number. Find the probability distribution.

\[
\begin{array}{cccccc}
1 & 2 & 3 & 4 & 5 & 6 \\
\frac{1}{4} & \frac{2}{4} & \frac{1}{5} & \frac{2}{5} & \frac{1}{6} & \frac{2}{6} \\
\end{array}
\]

\[p(2) = 2p(1)\quad \rightarrow \quad p(1) = p(4) = p(6) = 2x\]
\[p(4) = 2p(1)\quad \rightarrow \quad p(1) = p(3) = p(5) = x\]
\[p(\text{long even}) = x\]
\[p(\text{long odd}) = 2x\]

\[p(1) + p(2) + p(3) + p(4) + p(5) + p(6) = 1\]
\[x + 2x + x + 2x + x + 2x = 1\]
\[9x = 1\]
\[x = \frac{1}{9}\]

Example: A four sided die is rolled. If a four or a one is rolled the die is rolled a second time. The total sum of the numbers rolled is recorded. Give the probability distribution.

\[
\begin{array}{cccccccc}
2 & 3 & 4 & 5 & 6 & 7 & 8 \\
\frac{1}{4} & \frac{3}{16} & \frac{1}{16} & \frac{2}{16} & \frac{1}{16} & \frac{1}{16} & \frac{1}{16} \\
\end{array}
\]
Example: Use the probability distribution and the events to answer these questions.

<table>
<thead>
<tr>
<th>S</th>
<th>a</th>
<th>b</th>
<th>c</th>
<th>d</th>
<th>e</th>
<th>f</th>
</tr>
</thead>
<tbody>
<tr>
<td>prob</td>
<td>0.15</td>
<td>0.08</td>
<td>0.21</td>
<td>0.12</td>
<td>0.25</td>
<td>.19</td>
</tr>
</tbody>
</table>

\[ P(\phi) = 0 \quad P(\{f\}) = P(f) = .19 \]

\[ P(E) = 0.15 + 0.21 + 0.12 + 0.25 = .73 \]

\[ P(E^c) = 0.08 + 0.19 = 1 - 0.73 = 1 - P(E) \]

\[ P(F \cap G) = 0.08 + 0.12 \]

\[ P(E \cup G) = 1 - 0.19 \]

Must Know Probability Formulas:

\[ P(E \cup G) = P(E) + P(G) - P(E \cap G) \]

\[ P(E) = 1 - P(E^c) \]
Example: It is known from a survey that 29% of the people buy product A, 36% of the people buy product B and 11% buy both products. Find the probability that a person selected at random

A) buys only one of the product.

\[ \frac{.18}{.25} \]

B) doesn’t buy either of the products.

\[ .46 \]

Example: This table classifies the English, History, Math, and Poly Sci majors at State U according to their year. (There are no double majors.)

<table>
<thead>
<tr>
<th></th>
<th>Freshmen (F)</th>
<th>Sophomores(Soph)</th>
<th>Juniors (J)</th>
<th>Seniors (Sr)</th>
<th>Totals</th>
</tr>
</thead>
<tbody>
<tr>
<td>English (E)</td>
<td>64</td>
<td>35</td>
<td>31</td>
<td>11</td>
<td>171</td>
</tr>
<tr>
<td>History (H)</td>
<td>55</td>
<td>41</td>
<td>33</td>
<td>52</td>
<td>181</td>
</tr>
<tr>
<td>Math (M)</td>
<td>49</td>
<td>52</td>
<td>50</td>
<td>69</td>
<td>180</td>
</tr>
<tr>
<td>Poly Sci (PS)</td>
<td>70</td>
<td>33</td>
<td>41</td>
<td>57</td>
<td>181</td>
</tr>
<tr>
<td>Totals</td>
<td>218</td>
<td>141</td>
<td>155</td>
<td>199</td>
<td>713</td>
</tr>
</tbody>
</table>

If a student is selected at random, find the probability that

A) The student is a History major and a Sophomore.

\[ \frac{41}{713} \]

B) The student is not a Sophomore and is an English major.

\[ \frac{64 + 31 + 41}{713} = \frac{171 - 35}{713} \]

C) The student is a Math major or is a Senior.

\[ \frac{180 + 149 - 69}{713} = \frac{180 + 91 + 52 + 37}{713} \]
Example: Roll a 4 side die and a 6 sided die. Find the probability of

A) getting a sum greater than 9.

B) getting a sum of 6 and at least one die comes up a 2.

C) getting a sum of 6 or at least one die comes up a 2.