Section 6.3: The Multiplication Principle

An experiment is to flip a coin. How many outcomes are possible if the coin is flipped

A) Twice. \[ S = \{ \text{HH, HT, TH, TT} \} \]
\[ 4 = 2 \cdot 2 \]

B) Three times. \[ S = \{ \text{HHH, HHT, HTH, HTT, THH, THT, TTH, TTT} \} \]
\[ 8 = 2 \cdot 2 \cdot 2 \]
\[ 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 = 2^5 \]

C) Five times.

Example: An experiment is to draw two letters from a box that has an A, B, and C. How many outcomes are there?

\[ \begin{array}{ccc}
\text{Repeats} & \text{no repeats} \\
AA & AB & CA \\
AB & AC & CA \\
AC & BA & CB \\
BA & BC & CB \\
BC & CA & CB \\
BB & \text{in order} & \end{array} \]
\[ 9 = 3 \cdot 3 \]
\[ 6 = 3 \cdot 2 \]

Definition - Multiplication Principle: An outcome consists of \( k \) successive selections with \( n_i \) choices for the \( i \)-th selection. The total number of outcomes is

\[ n_1 \cdot n_2 \cdot n_3 \cdots n_k = \prod_{i=1}^{k} n_i \]

Example: There are 6 roads from town A to town B and 7 roads from town B to town C. How many ways can you go from town A to town C?

\[ 6 \cdot 7 \]
\[ A \rightarrow B \; B \rightarrow C \]

\[ \frac{6 \cdot 6 \cdot 6 \cdot 7}{A \rightarrow B \; AA \; AB \; BC} \]
Example: How many ways can you select a president, vice-president and secretary from a group of 10 people?

\[ \frac{10 \cdot 9 \cdot 8}{\cancel{9} \cdot \cancel{8}} \]

**Definition:** A **factorial**, \( n! \), is the product of integers from \( n \) down to 1. For example: \( 5! = 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 \).

By definition, \( 0! = 1 \)

\[ 3! = 3 \cdot 2 \cdot 1 \]

Example: Compute the following.

A) \( 10! \)

B) \( 14! \)

Example: How many three digit numbers can be formed from the digits: \( 2, 3, 4, 5, 6, 7, 8 \).  

A) No restrictions.

B) The number is even.

C) The digits are even.

D) The number is even and no digit is repeated.
Example: Five boys and five girls are to be seated in a row. Find how many ways can this be done if

A) no restrictions.

\[10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 10!\]

B) they alternate seats.

\[2 \left( \frac{5 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 2 \cdot 1 \cdot 1}{b \cdot y \cdot b \cdot y \cdot b \cdot y \cdot b \cdot y} \right) = 2 \cdot 5! \cdot 5!\]

C) girls sit together and boys sit together.

\[2 \cdot \left( \frac{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{9} \right) \cdot 2 \cdot 5! \cdot 5!\]

D) girls sit together.

\[6 \cdot \left( \frac{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{9} \right) \cdot 3 \cdot 5\]
E) Sue, Jill, or Sarah are seated in the end seats.

$$\underline{3 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 \cdot 2}$$

$$3 \cdot 2 \cdot 8!$$

Example: How many 3 digit numbers have none of the digit being a 7.

B) Exactly one digit a 7.

$$\frac{1 \cdot 9 \cdot 9}{4} + \frac{8 \cdot 1 \cdot 9}{4} + \frac{8 \cdot 9 \cdot 1}{4}$$

D) Exactly three digits a 7.

$$\frac{1 \cdot 9 \cdot 1}{1^4} + \frac{1 \cdot 9 \cdot 1}{1^4} + \frac{8 \cdot 1 \cdot 1}{1^4}$$

E) no digits repeated and the number is even.
Example: How many 5 digit numbers have at least one digit being a 7?

\[
\text{want} = \text{Total} - \text{don't want}
\]

\[
\frac{9 \cdot 10 \cdot 10 \cdot 10 \cdot 10}{9 \cdot 9 \cdot 9 \cdot 9 \cdot 9}
\]

Example: A computer code is to be constructed with either 5 letters or 2 letters followed by three digits. How many codes are possible if no letters may be repeated in the code.

\[
\text{all letters} \quad 26 \cdot 25 \cdot 24 \cdot 23 \cdot 22
\]

\[
\text{2 letters + 3 digits} \quad 26 \cdot 25 \cdot 10 \cdot 10 \cdot 10
\]
Example: Four couples are to be seated in a row. How many ways can this be done if the couples are to be seated together?

\[
c_1 \cdot c_2 \cdot c_3 \cdot c_4
\]

\[
\left(4 \cdot 3 \cdot 2 \cdot 1\right) \cdot 2^4
\]

Example: An ATM code contains 4 digits. How many codes are possible if the bank will not allow the codes to have all the same digits?

Bank vetoed 10 codes.

\[
\text{Total} = 10 \cdot 10 \cdot 10 \cdot 10 = 10^4
\]

\[
10^4 - 10
\]