Section 6.1: Sets and Set Operations

**Definition:** A set is a well defined collection of objects. The objects in a set are called the elements of the set.

- **roster notation**
  \[ A = \{a, b, c\} \]
  \[ \{b, c, a\} \]
  \[ b \in A \]
  \[ j \notin A \]

- **set-builder notation**
  \[ B = \{x \mid x \text{ is a student in Math 150.501} \} \]

**Definition:** The **empty set** is the set that doesn’t have any elements, denoted by \( \emptyset \) or \( \{\} \). The **universal set** is the set that contains all of the elements for a problem, denoted by \( U \).

**Definition:** Subset: \( J \subseteq K \) if for all \( x \in J \) then \( x \in K \)

**Definition:** Proper Subset: \( J \subset K \) if for all \( x \in J \) then \( x \in K \) and \( J \neq K \)

Example: If \( J \) is any set, is \( \emptyset \subseteq J \)? **yes.**

is \( J \subseteq J \)? **yes**
Example: Give all the subsets for these sets.

A) \( A = \{1, 2\} \)

\[ A \] = \{\{1\}, \{2\}, \{1, 2\}, \emptyset\}

4 subsets. 3 proper subsets

B) \( B = \{a, b, c\} \)

\[ B \] = \{\{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}, \emptyset\}

8 subsets. 7 proper subsets

Example: How many subsets does the set \( K \) have? How many proper subsets does \( K \) have?

\[ K = \{a, b, c, d, e, f, g, h\} \]

\[ n(K) = 8 \]

\[ \text{# of elements in } K \]

\[ \text{# of subsets} = 2^{n(K)} = 2^8 \]

\[ \text{# of proper subsets} = 2^{n(K)} - 1 = 2^8 - 1 \]

Set Quiz

Circle the correct answer.

\( A = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\} \)

\( B = \{1, 2, 3, 5, 8\} \)

\( C = \{2, 3, 5, 7, 8\} \)

\( D = \{1, 2, 5\} \)

\( E = \{1, 2, 5\} \)

\( F = \{\emptyset, 7, 8\} \)

\( G = \{1, 2, \{1, 2\}\} \)

- True  False  \( 2, 7 \in A \)
- True  False  \( C \subseteq B \)
- True  False  \( \{1, 2, 3\} \subseteq A \)
- True  False  \( 5 \in D \)
- True  False  \( 1 \subseteq D \)
- True  False  \( \{2, 5\} \subseteq D \)
- True  False  \( \{1, 2, 3\} \subseteq \emptyset \)
- True  False  \( \{2, 5\} \subseteq D \)
- True  False  \( \{1, 2\} \subseteq G \)
- True  False  \( \emptyset \in A \)
- True  False  \( \emptyset \in F \)
- True  False  \( \emptyset = 0 \)
- True  False  \( \emptyset = \{\emptyset\} \)
Set Operations

- Intersection: $A \cap B = \{x | x \in A \text{ and } x \in B\}$
- Union: $A \cup B = \{x | x \in A \text{ or } x \in B\}$
- Compliment: $A^C = A' = \{x | x \in U \text{ and } x \notin A\}$

Example: Use these sets to answer the following.

$U = \{0, 1, 2, \ldots, 9\}$  
$A = \{0, 2, 4, 6, 8\}$  
$B = \{1, 3, 5, 7, 9\}$  
$F = \{1, 3, 8, 9\}$

$C = \{0, 3, 4, 5, 7\}$  
$D = \{4, 5, 6, 7, 8, 9\}$  
$E = \{0, 6, 7, 9\}$

A) $B \cap D = \{7, 9\}$

B) $E \cap F \cap A = \emptyset$  

C) $B \cup E = \{0, 1, 2, 3, 4, 5, 6, 7, 9\}$

D) $A \cup B = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$
\[ U = \{0, 1, 2, \ldots, 9\} \quad A = \{0, 2, 4, 6, 8\} \quad B = \{1, 3, 5, 7, 9\} \quad F = \{1, 3, 8, 9\} \]
\[ C = \{0, 3, 4, 5, 7\} \quad D = \{4, 5, 6, 7, 8, 9\} \quad E = \{0, 6, 7, 9\} \]

E) \( F^c = \{0, 2, 4, 5, 6, 7\} \)

F) \( (D \cup E)^c \cap (C \cup F) = \{1, 3\} \)

\[ C \cup F = \{0, 1, 3, 4, 5, 7, 8, 9\} \]
\[ D \cup E = \{0, 4, 5, 6, 7, 8, 9\} \]
\[ (D \cup E)^c = \{1, 2, 3\} \]

Example: Use the given sets to express the set operations in words.

Let \( U \) denote all students at Texas A\&M
\[ B = \{x \in U | x \text{ lives in Bryan}\} \]
\[ C = \{x \in U | x \text{ lives in College Station}\} \]
\[ S = \{x \in U | x \text{ is a sophomore}\} \]

A) \( B^C \cap S \) The students who do not live in Bryan are soph.

The soph that do not live in Bryan.

B) \( C^C \cup S \) The students who do not live in C.S. combined with the students who are soph.

C) \( (B \cup C) \cap S \) The soph that live in Bryan or C.S.
Venn Diagrams

A Venn Diagram is a pictorial way of representing a set.

Example: Shade the parts of a Venn diagram that represents these set operations.

A) $A \cap B$

B) $(A \cup B) \cap C$

Answer

$A = \{1,2,4,5\}$

$B = \{2,3,5,6\}$

$C = \{4,5,6,7\}$

$U = \{1, \ldots, 8\}$

$C^c = \{1,2,3,8\}$

$B \cup C^c = \{1,2,3,5,6,8\}$

$A \cap (B \cup C^c) = \{1,2,5\}$
Example: Write an expression both with set operations and with words that describes the shaded region of the Venn Diagram.
Use the given sets to express the set operations in words.

Let $U$ denote all students at Texas A& M  
$B = \{x \in U | x \text{ plays basketball} \}$  
$T = \{x \in U | x \text{ plays tennis} \}$  
$G = \{x \in U | x \text{ plays golf} \}$

\[ T^c \cap (B \cup C) \]

The basketball & golf players who do not play tennis.

\[ (T \cap B \cap C^c) \cup (T \cap B \cap C) \cup (T^c \cap B \cap C) \]

people who play exactly 2 sports.

\[ (T \cap A \cap B^c) \cup (T \cap A \cap B) \cup (T^c \cap A \cap B) \]
Definition: Sets A and B are said to be disjoint if

\[ A \cap B = \emptyset \]