Section 5.1: Simple Interest and Compound Interest

Definition: If the principal, \( P \), is invested for a time period of \( t \) at a simple interest rate of \( r\% \) (for that period) then the interest earned at the end of the time period is given by

\[
I = Prt
\]

The future value, \( A \) or \( F \) of the investment at the end of the period is

\[
A = P + I = P(1 + rt)
\]

Example: You invest $500 at an annual simple interest rate of 4% for 6 years. How much interest did you earn? What is the balance at the end?

\[
I = Prt = 500(0.04)(6) = 120
\]

\[
A = P(1 + rt) = 500(1 + (0.04)(6)) = 620
\]

Example: You invest $1000 at a monthly simple interest rate of 6.5% for 2 years. How much interest did you earn? What is the balance at the end?

\[
I = Prt = 1000(0.065)(24) = 1560
\]

\[
A = 1000 + 1560 = 2560
\]
Example: You invest $2000 for 8 months and at the end of this time period you have earned $400 of interest. What is the annual simple interest rate? monthly simple interest rate?

\[ I = P \times r \times t \]
\[ 400 = 2000 \times r \times \left( \frac{8}{12} \right) \]
\[ r = 3.0\% \]

Annual.

\[ I = P \times r \times t \]
\[ 400 = 2000 \times r \times \left( \frac{1}{12} \right) \]
\[ r = 2.5\% \]

Monthly.

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Definition: Suppose the principal, \( P \), is invested for \( t \) years at an annual interest rate of \( r\% \) and interest is compounded \( m \) times per year. The future amount, \( A \) or \( F \), is given by

\[ A = P(1 + i)^n = P \left( 1 + \frac{r}{m} \right)^{mt} \]

\[ i = \frac{r}{m} \leq \text{rate per period} \]

\[ n = mt \leq \text{total # of compoundings} \]
Example: Find the balance of the account if you invest $600 for 7 years at a nominal rate of 5% compounded

A) annually.

\[ A = 600 \left(1 + \frac{0.05}{1}\right)^7 = 844.26 \]

B) semiannually.

\[ 600 \left(1 + \frac{0.05}{2}\right)^{2(7)} = 847.78 \]

C) quarterly.

\[ 600 \left(1 + \frac{0.05}{4}\right)^{4(7)} = 849.60 \]

D) monthly.

\[ 600 \left(1 + \frac{0.05}{12}\right)^{12(7)} = 8570.82 \]

E) daily.

\[ 600 \left(1 + \frac{0.05}{365}\right)^{365(7)} = 851.42 \]

Example: You want $2000 in an account at the end of 3 years. If the account gets a nominal rate of 5.75% compounded quarterly, how much do you start the account with?

\[ A = P \left(1 + \frac{r}{m}\right)^{mt} \]

\[ \frac{A}{\left(1 + \frac{r}{m}\right)^{mt}} = P = \frac{2000}{\left(1 + \frac{0.0575}{4}\right)^{4(3)}} \]

\[ P = \$1685.18 \]
Example: You have the choice of investing money in one of two different accounts. The first account is at Bank A and has a rate of 6.51% compounded semiannually. The second account is at Bank B and has a rate of 6.08% compounded daily. Which account is the better deal?

\[
\frac{A}{r_{eff} = 100 \left( 1 + \frac{0.0451}{2} \right)^{2} \cdot 100} = 6.61595 \%
\]

\[
\frac{B}{r_{eff} = 0.0608 \text{ or } 6.08\%}
\]

**Definition:** For compound interest, the effective yield, \( r_{eff} \), is given by

If this account earned a simple interest rate for 1 yr → \( r_{eff} \)

\[
r_{eff} = 100 \left( 1 + \frac{r}{m} \right)^{m} \cdot 100
\]

Example: You invest $2000 in an account that pays interest compounded monthly. What interest rate do you need to have a balance of $5000 at the end of 3 years.

\[
A = P \left( 1 + \frac{r}{m} \right)^{mt}
\]

\[
5000 = 2000 \left( 1 + \frac{r}{12} \right)^{36}
\]

\[
2.5 = \frac{5000}{2000} = \left( 1 + \frac{r}{12} \right)^{36}
\]

\[
2.5 = \left( 1 + \frac{r}{12} \right)^{36}
\]

\[
(2.5)^{\frac{1}{36}} = 1 + \frac{r}{12}
\]

\[
(2.5)^{\frac{1}{36}} - 1 = \frac{r}{12}
\]

\[
r = 12 \left[ (2.5)^{\frac{1}{36}} - 1 \right]
\]

\[
r = 30.9359\%
\]
TVM Solver

The TVM solver that is built function on the TI-83/84 calculators. If you are using the old TI-83 press 2nd [x⁻¹] and then press [ENTER], otherwise press the [APPS] and select the Finance application and press enter. Here are the variables that are used in the TVM Solver.

\[ N = m \times t \] which is the total number of periods (compoundings) for the life of the account.

\[ I\% = \] The interest rate per year as a percentage.

\[ PV = \] The present value (starting value) of the account.

\[ PMT = \] This is the payment that is made each period.

\[ FV = \] The future value (end value) of the account.

\[ P/Y = \] The number of payments per year.

\[ C/Y = \] The number of compoundings per year.

For this class, P/Y and C/Y are equal and **PMT:END BEGIN** should be set to END.