Section 3.1: Graphing Systems of Linear Inequalities

Definition: The Feasible Region (FR), sometimes called the solution set, for a system of inequalities are all the points $(x,y)$ that satisfies all of the inequalities at the same time. The feasible region is usually illustrated graphically with the $xy$-plane.

Example: Sketch the feasible region for these inequalities/system of inequalities.

A) $3x + 2y > 6$

$$3x + 2y = 6$$

<table>
<thead>
<tr>
<th>$x$</th>
<th>$y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>3</td>
</tr>
<tr>
<td>2</td>
<td>0</td>
</tr>
</tbody>
</table>

Example: Sketch the feasible region for these inequalities/system of inequalities.

B) $3x - y \geq 0$

<table>
<thead>
<tr>
<th>$x$</th>
<th>$y$</th>
</tr>
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<tbody>
<tr>
<td>0</td>
<td>0</td>
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<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>3</td>
</tr>
</tbody>
</table>

$3 - y = 0$

$3 > y$
Example: Sketch the feasible region for these inequalities/system of inequalities.

\[ \begin{align*}
& (1) \quad 3x + 2y \leq 16 \\
& (2) \quad x - y \leq 2 \\
& (3) \quad y \geq 0 \\
& (4) \quad x \geq 0 \\
& (5) \quad x \leq 8 \\
& (6) \quad y \leq 4 \\
& (7) \quad x \geq 0 \\
& (8) \quad y \geq 0 \\
& (9) \quad x \leq 8 \\
& (10) \quad y \leq 4 \\
\end{align*} \]
Definition: A feasible region is said to be bounded if it can be enclosed by a circle. Otherwise, it is unbounded.

Definition: The intersection of two inequalities (if possible), is called a corner point of a feasible region provided that this point is part of the feasible region.

Example: Find the corner points for part C of the last example.

Example: Determine the feasible region for this system of inequalities. Find all corner points and determine if the feasible region is bounded.

\begin{align*}
\text{1. } & x + y \leq 2 \\
\text{2. } & 3x + y \geq 6 \\
\text{3. } & x + 3y \geq 6 \\
\end{align*}

With Solution.
Example: Determine the feasible region for this system of inequalities. Find all corner points and determine if the feasible region is bounded.

\[
\begin{align*}
&x + y \leq 6 \\
x + y \geq 2 \\
2x - y \geq 0 \\
x \leq 5 \\
y \leq 6 \\
0 < x \leq 2 \\
0 \leq y < 2 \\
0 \leq x < 3 \\
0 < y \leq 3 \\
\end{align*}
\]

A \left( \frac{5}{2}, \frac{5}{2} \right) \\
B \left( \frac{5}{2}, 2 \right) \\
C \left( \frac{5}{2}, 0 \right) \\
D (5, 0) \\
E (2, 0) \\

\begin{align*}
x + y &= 6 \\
2x - y &= 0 \\
x + y &= 2 \\
2x - y &= 0
\end{align*}