Section 2.1: Systems of Linear Equations

Example: Set up the following word problems. Do not solve (yet). Be sure that the variables are defined.

1. You own a hamburger stand and your current inventory includes 86 bread rolls, 100 beef patties, and 140 cheese slices. Your menu consists of three types of hamburgers: plain, double cheeseburger, and regular cheeseburger. Each plain hamburger requires 1 beef patty and 1 bread roll. Each double cheeseburger requires 1 bread roll, 2 beef patties, and 4 cheese slices. Each regular cheeseburger requires 2 cheese slices, 1 bread roll, and 1 beef patty. How many of each hamburger should you make so that all resources are used?

\[
\begin{array}{c|c|c|c}
& \text{Plain} & \text{Double} & \text{Reg} \\
\hline
\text{Bread Roll} & 1 & 1 & 1 \\
\text{Beef Patties} & 1 & 2 & 1 \\
\text{Cheese Slices} & 0 & 4 & 2 \\
\end{array}
\]

\[
\begin{align*}
4y + 2z &= 140 \\
2x + 2y + z &= 100 \\
x + y + z &= 86
\end{align*}
\]

Answer:

3. The city of Aurora gets a matching grant of $1 million (making a total of $2 million) to spend in these categories: streets, sewers, and parks. The city council decides that the amount spent on parks should equal the total amount spent on streets and sewers and that, furthermore, twice as much should be spent on parks as on sewers. How much money is to be allotted to each of these categories?

\[
\begin{align*}
x &= \text{spent on streets in millions} \\
y &= \text{spent on sewers} \\
z &= \text{spent on parks} \\
\hline
\end{align*}
\]

\[
\begin{align*}
x + y + z &= 2 \\
z &= x + y \\
z &= 2y
\end{align*}
\]

\[
\begin{array}{c|c|c}
\text{Parks} & \text{Sewers} & \text{Streets} \\
\hline
20 & 10 & 100 \\
100 & 50 & \end{array}
\]

\[
\frac{1}{2}z = y
\]
4. Link has $7,000 to invest. He decides to invest in three different companies. The QX company costs $25 per share and pays dividends of $0.75 per share each year. The RY company costs $40 per share and pay dividends of $1.50 per share each year. The KZ company costs $15 per share and pays $2.00 per share per year in dividends. Link wants to have twice as much money in the RY company as in the KZ company. Link also wants to earn $352 in dividends per year. How much should Link invest in each company to meet his goals?

\[ \begin{align*}
X &= \text{number of shares in QX} \\
Y &= \text{number of shares in RY} \\
Z &= \text{number of shares in KZ} \\
X + Y + Z &= 7000 \\
Y &= 2Z \\
0.75 \left( \frac{X}{25} \right) + 1.50 \left( \frac{Y}{40} \right) + 2 \left( \frac{Z}{15} \right) &= 352
\end{align*} \]

<table>
<thead>
<tr>
<th>RY</th>
<th>KZ</th>
</tr>
</thead>
<tbody>
<tr>
<td>20</td>
<td>10</td>
</tr>
<tr>
<td>Y</td>
<td>2Z</td>
</tr>
</tbody>
</table>

\[ Y = 2Z \]

\[ \begin{align*}
25X + 40Y + 15Z &= 7000 \\
0.75X + 1.50Y + 2Z &= 352 \\
40Y = 2(15Z) \rightarrow 40Y = 30Z
\end{align*} \]
5. A brokerage firm packaged blocks of blue-chip stocks, bonds and high-risk stocks into three portfolios, which it offers to its customers. The makeup of each portfolio is given in the table. A customer wants eight blocks of blue-chip stocks, 11 blocks of bonds, and nine blocks of high-risk stocks. How many of each portfolio should the customer purchase?

<table>
<thead>
<tr>
<th>Portfolio</th>
<th>blue chip stocks</th>
<th>bonds</th>
<th>high-risk stocks</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>1 block</td>
<td>4 blocks</td>
<td>3 blocks</td>
</tr>
<tr>
<td>II</td>
<td>2 blocks</td>
<td>1 block</td>
<td>2 blocks</td>
</tr>
<tr>
<td>III</td>
<td>4 blocks</td>
<td>2 blocks</td>
<td>1 block</td>
</tr>
</tbody>
</table>

\[
\begin{align*}
    x &= \text{blocks of portfolio I bought} \\
    y &= \text{blocks of portfolio II bought} \\
    z &= \text{blocks of portfolio III bought}
\end{align*}
\]

\[
\begin{align*}
    x + 2y + z &= 8 \\
    4x + y + 2z &= 11 \\
    3x + 2y + z &= 9
\end{align*}
\]