Section 8.2: Expected Value

Example: A game cost $2 to play. The game consists of rolling two fair die. If you get a sum of 2 or 3, then you win $50. A sum of 4 wins $10. Every other sum constitutes a loss of the game. Let \( X \) be the player’s net winnings.

A) Find the probability distribution of \( X \).

\[
\begin{array}{c|ccc}
X & 4 & 8 & -2 \\
\text{prob} & \frac{3}{36} & \frac{2}{36} & \frac{30}{36}
\end{array}
\]

B) Would you rather play this game or run the game?

\[
\text{E}(x) = \left( \frac{3}{36} \right) \times 4 + \left( \frac{2}{36} \right) \times 8 + \left( \frac{30}{36} \right) \times (-2) = \frac{3}{3}
\]

C) If this game is played 1 million times, what results would be expected?

\[
\text{player would win} \quad \frac{3}{3} \text{ million.}
\]

\[
\begin{array}{cccccc}
1 & 2 & 3 & 4 & 5 & 6 \\
1 & X & X & X & & \\
2 & X & & & & \\
3 & & & & & \\
4 & & & & & \\
5 & & & & & \\
6 & & & & & \\
\end{array}
\]
**Definition:** The expected value or mean of a discrete random variable X, denoted E(X) is

\[ E(x) = x_1 \cdot p_1 + x_2 \cdot p_2 + x_3 \cdot p_3 + \ldots + x_n \cdot p_n \]

Long term result: \( E(x) = 0 \)

Note: A game is said to be fair when \( E(x) = 0 \)

**Example:** A casino has a game where the player draws two different cards from a standard deck of cards. If both cards are diamonds, then the player wins $5. One diamond drawn means the player wins $1. What should be charged in order to make the game fair?

<table>
<thead>
<tr>
<th>( X )</th>
<th>( 5 - A )</th>
<th>( 1 - A )</th>
<th>( -A )</th>
</tr>
</thead>
<tbody>
<tr>
<td>prob</td>
<td>( \frac{2}{34} )</td>
<td>( \frac{13}{34} )</td>
<td>( \frac{19}{34} )</td>
</tr>
</tbody>
</table>

\[ E(x) = \begin{cases} 2 \cdot \frac{2}{34} + 1 \cdot \frac{13}{34} + 0 \cdot \frac{19}{34} \end{cases} \]

\[ 0 = 2 \cdot (5 - A) + 13 \cdot (1 - A) + 19 \cdot (-A) \]

\[ 0 = 2 - 2A + 13 - 13A - 19A \]

\[ 0 = 23 - 34A \]

\[ 34A = 23 \]

\[ A = \frac{23}{34} \approx 0.67647 \]

\[ A \approx 0.68 \]
Example: A company sells one year term life insurance policies for $800. The face value of the policy is $25,000. Life insurance tables have determined that the probability that a person interested in this policy will survive the year is 0.97. What is the company's expected profit on this product?

\[ X = \text{companies not profit:} \]

\[
\begin{array}{c|cc}
X & \text{Life} & \text{not} \\
\hline
800 & -24200 \\
\hline
\text{prob} & 0.97 & 0.03
\end{array}
\]

\[ E(x) = 800(0.97) + (-24200)(0.03) \]
\[ = 850 \]

**Definitions:** The odds that an event E will occur, odds in favor of E, are given as a to b or a:b where a and b are integers and the fraction a/b is in reduced form.

For every 3 times E happens there are 8 times E does not happen.

Example: The odds in favor of the event E are 3 to 8.

A) Find the odds against E. \[ 8 \text{ to } 3 \]

B) Find \( P(E) \) and \( P(E^c) \).

\[ P(E) = \frac{3}{3+8} = \frac{3}{11} \]
\[ P(E^c) = \frac{8}{3+8} = \frac{8}{11} \]
Example: The odds in favor of Joe telling a funny joke (in class) is 12 to 5. Find the probability of Joe telling a funny joke.

\[
\frac{12}{12+5} = \frac{12}{17}
\]

Computing odds from probability

1. \( \frac{P(A)}{P(A^c)} \)

2. Reduce to \( \frac{c}{d} \)

3. Answer: \( \frac{c}{d} \)

Example: If \( P(E) = 0.18 \), find the odds in favor of \( E \).

\[
\frac{0.18}{1 - 0.18} = \frac{9}{41}
\]

Answer: \( \frac{9}{41} \)
Example: If $P(F) = 0.32$, find the odds against $F$.

\[
\frac{.32}{.68} = \frac{8}{17}
\]

Answer: $17 \to 8$

Definition: The mean of a data set is the traditional average.

The median of a data set is the middle of the data when the data is ordered. The median is sometime called the second quartile which means that at least 50% of the data is that number or greater.

The mode of a data set are the values that have the largest frequencies.

Example: Compute the mean, median, and mode of these data values.

\[90, 70, 85, 55, 60, 65\]

\[50, 50, 60, 65, 85, 85, 90\]

\[
\begin{align*}
\text{Mean} &= 69.2857 \\
\text{Median} &= 65 \\
\text{Mode} &= 85
\end{align*}
\]
Example: Compute the mean, median, and mode of these data values.

50, 65, 70, 75, 90, 95

\[ \text{Mean} = \frac{70 + 75}{2} = 72.5 \]

\[ \text{Mode} = \left\{ \begin{array}{l} \text{no mode} \\ \text{all the data values.} \end{array} \right. \]
Example: Compute the mean, median, and mode of this probability distribution.

<table>
<thead>
<tr>
<th>X</th>
<th>1</th>
<th>3</th>
<th>4</th>
<th>6</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>prob.</td>
<td>0.2</td>
<td>0.15</td>
<td>0.1</td>
<td>0.3</td>
<td>0.25</td>
</tr>
</tbody>
</table>

Mode = 6

\[ E(x) = \text{Mean} = 4.6 \]

Median = 6

Example: The distribution of grades on an exam are given in the table. Discuss the values for the mean, median, and mode.

<table>
<thead>
<tr>
<th>grades</th>
<th>90 ≤ x ≤ 99</th>
<th>80 ≤ x ≤ 89</th>
<th>70 ≤ x ≤ 79</th>
<th>60 ≤ x ≤ 69</th>
<th>50 ≤ x ≤ 59</th>
<th>40 ≤ x ≤ 49</th>
</tr>
</thead>
<tbody>
<tr>
<td>Freq.</td>
<td>8</td>
<td>15</td>
<td>18</td>
<td>11</td>
<td>5</td>
<td>2</td>
</tr>
</tbody>
</table>

Mode (category): 70's.

\[ \overline{x} = 75.67796 \]
Example: Explain the difference between a *population* and a *sample*.

Everybody

[...]

Example: A class of 100 students is divided into 4 lab sections: A, B, C, and D. After the first exam, the averages for the first three sections were 75.5, 64.2 and 68 respectively and the average for the entire class was 73.8. Do these numbers represent a sample or a population?