Section 8.1: Distribution of Random Variables

**Definition:** A **random variable** is a rule that assigns a real number to each outcome of a sample space.

Example: Let $X$ be the number of boys in a 3 kid family.

$S = \{ \text{bbb, bbg, bgb, gbb, ggb, gbgb, bgbb, ggg} \}$

\[ 3 \ 2 \ 2 \ 2 \ 1 \ 1 \ 1 \ 0 \]

A) What are the values of the random variable $X$?

\[ 0, 1, 2, 3 \]

B) Give the probability distribution for $X$.

\[
\begin{array}{c|c|c|c|c}
X & 0 & 1 & 2 & 3 \\
\hline
\text{prob.} & \frac{1}{8} & \frac{3}{8} & \frac{3}{8} & \frac{1}{8} \\
\end{array}
\]

Types of Random Variables

1) **Continuous R.V.**
   - **Don't skip values.**
   - **Example:** time, distance, weight, ...

2) **Discrete R.V.**
   - **Skip values.**
   - a) **Finite**
     - **Fixed # of values.**
   - b) **Infinite**
     - **Infinite # of values.**
Example: Classify these random variables. Give the values of the random variable.

A) \( X = \) the number of hours you sleep in a day.
\[
\text{cont.} \quad 0 \leq x \leq 24
\]

B) \( X = \) the number of good jokes/puns that I tell in a semester.
\[
\text{discrete (finite)} \quad 0, 1, 2, \ldots \quad \text{Max #}
\]

C) \( X = \) the number of rolls it takes to get a 5 on a 10-sided die.
\[
\text{discrete (infinite)} \quad 1, 2, 3, \ldots
\]

D) \( X = \) the number of yellow balls drawn in a sample of 6 from a box that contains 5 yellow balls, 2 green balls, 1 red ball and 1 purple ball.
\[
\text{discrete} \quad 2, 3, 4, 5
\]

Example: Let \( X = \) the number of clubs in a three card hand.

Compute \( P(X = 2) \): \[
\frac{\binom{13}{2} \cdot \binom{39}{1}}{\binom{52}{3}}
\]
**Definition:** A **histogram** is a way to present the probability distribution of a discrete random variable.

Example: Draw the probability distribution $X$.

<table>
<thead>
<tr>
<th>$X$</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>prob.</td>
<td>0.2</td>
<td>0.1</td>
<td>0.15</td>
<td>0.4</td>
<td>0.15</td>
</tr>
</tbody>
</table>

Base = 1
Rect. centered on the value $X$

height = prob.

Example: The following histogram is only missing the rectangle at $x = 7$.

A) Find $P(X = 7) = .15$

B) Give the probability distribution for $X$.

C) Find $P(2 \leq X < 6) = .1 + .25 + .1$
Example: A cookie company wants to check the consistency of the number of raisins in its oatmeal raisin cookies. A few cookies from each batch are selected and the number of raisins are counted. After several days, the following results were found:

<table>
<thead>
<tr>
<th>number of cookies</th>
<th>3</th>
<th>5</th>
<th>6</th>
<th>8</th>
<th>freq.</th>
<th>total = 22</th>
</tr>
</thead>
<tbody>
<tr>
<td>number of raisins</td>
<td>2</td>
<td>7</td>
<td>9</td>
<td>10</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Define the random variable and give the probability distribution.

\[ X = \# \text{ of Raisins.} \]

<table>
<thead>
<tr>
<th></th>
<th>2</th>
<th>7</th>
<th>9</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>prob</td>
<td>(\frac{3}{22})</td>
<td>(\frac{5}{22})</td>
<td>(\frac{6}{22})</td>
<td>(\frac{8}{22})</td>
</tr>
</tbody>
</table>