Section 7.2: Definition of Probability

**Definition:** Probability is a number that is assigned to an outcome of a sample space that indicates how likely that outcome is to happen when conducting an experiment.

\[
P(r) > P(g) > P(c)
\]

**Rules of Probability**

1) \[0 \leq \text{all prob} \leq 1\]

2) \[\text{all prob adds up to 1}\]

<table>
<thead>
<tr>
<th>Die Roll</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Method A</td>
<td>1/6</td>
<td>1/6</td>
<td>1/6</td>
<td>1/6</td>
<td>1/6</td>
<td>1/6</td>
</tr>
<tr>
<td>Method B</td>
<td>0.1</td>
<td>0.2</td>
<td>0.3</td>
<td>0.2</td>
<td>0.1</td>
<td>0.3</td>
</tr>
<tr>
<td>Method C</td>
<td>0.1</td>
<td>0.2</td>
<td>0.3</td>
<td>0.2</td>
<td>0.1</td>
<td>0.1</td>
</tr>
</tbody>
</table>

Definition: A sample space where every outcome has the same probability (chance of happening) is called a uniform sample space and

\[P(\text{any individual outcome}) = \frac{1}{n(S)}\]

Definition: A probability distribution is a chart that shows the probability of every outcome in the sample space.

Example: A group of people were asked what was their favorite soft drink. The results of the survey are given in the table.

<table>
<thead>
<tr>
<th>drinks</th>
<th>Dr Pepper</th>
<th>Coke</th>
<th>Pepsi</th>
<th>Rootbeer</th>
<th>other</th>
</tr>
</thead>
<tbody>
<tr>
<td>number of people</td>
<td>175</td>
<td>10</td>
<td>40</td>
<td>60</td>
<td>25</td>
</tr>
</tbody>
</table>

Give the probability distribution that is associated with this survey.
Example: The number of grades for a group of Math 166 students are shown in the table.

<table>
<thead>
<tr>
<th>grade</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>F</th>
</tr>
</thead>
<tbody>
<tr>
<td>frequency</td>
<td>40</td>
<td>84</td>
<td>71</td>
<td>38</td>
<td>31</td>
</tr>
</tbody>
</table>

Total 264

If a student from this group is selected at random, what is the probability that the student made

A) an A?

\[
\frac{40}{264}
\]

B) an A or a B?

\[
\frac{40 + 84}{264}
\]

Example: You flip a coin 10, 50, and then 100 times. For each of these, how many times will you get a head?

Definition: If \( E \) is an event of a sample space, then

\[
P(E) = \text{add all the prob. of the outcomes in } E.
\]

\[
P(S) = 1 \quad P(\emptyset) = 0
\]

If \( S \) is uniform,

Then \( P(E) = \frac{n(E)}{n(S)} = \frac{n(E)}{n(S)} \)
Example: A single card is drawn from a standard deck of cards. Find these probabilities.

A) $P(\text{king}) = \frac{4}{52}$

B) $P(\text{heart}) = \frac{13}{52}$

C) $P(\text{heart or a king}) = \frac{13 + 4 - 1}{52} = \frac{16}{52}$

D) $P(\text{not a king or not a queen}) = \frac{1}{1}$

Example: Roll two six-sided dice (one red and one green)

A) Give the probability distribution for the sum of the dice.

B) Find the probability that the sum of the dice is a multiple of 5.
Example: Roll a 6 sided die. Suppose that any even number is twice as likely to happen as any odd number. Find the probability distribution.

<table>
<thead>
<tr>
<th></th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>prob</td>
<td>$\frac{1}{6}$</td>
<td>$\frac{1}{6}$</td>
<td>$\frac{1}{6}$</td>
<td>$\frac{1}{6}$</td>
<td>$\frac{1}{6}$</td>
<td>$\frac{1}{6}$</td>
</tr>
</tbody>
</table>

\[ P(2) = 2P(1) \quad P(1) = P(3) = P(5) = X \]
\[ P(2) = P(4) = P(6) = 2X \]

\[ P(1) + P(2) + P(3) + P(4) + P(5) + P(6) = 1 \]
\[ X + 2X + X + 2X + X + 2X = 1 \]
\[ 9X = 1 \]
\[ x = \frac{1}{9} \]

Example: A four sided die is rolled. If a four or a one is rolled the die is rolled a second time. The total sum of the numbers rolled is recorded. Give the probability distribution.

<table>
<thead>
<tr>
<th></th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>prob</td>
<td>$\frac{5}{16}$</td>
<td>$\frac{5}{16}$</td>
<td>$\frac{1}{16}$</td>
<td>$\frac{2}{16}$</td>
<td>$\frac{1}{16}$</td>
<td>$\frac{1}{16}$</td>
<td>$\frac{1}{16}$</td>
</tr>
</tbody>
</table>
Example: Use the probability distribution and the events to answer these questions.

<table>
<thead>
<tr>
<th>S</th>
<th>E</th>
<th>E</th>
<th>E</th>
<th>E</th>
<th>E</th>
</tr>
</thead>
<tbody>
<tr>
<td>prob</td>
<td>0.15</td>
<td>0.08</td>
<td>0.21</td>
<td>0.12</td>
<td>0.25</td>
</tr>
</tbody>
</table>

$E = \{ a, c, d, e \}$ $F = \{b, d, f\}$ $G = \{a, b, d\}$

A) $P(S) = 1$ $P(\emptyset) = 0$ $P(f) = P(\{f\}) = 0.19$

B) $P(E) = 0.15 + 0.21 + 0.12 + 0.25 = 0.73$

C) $P(E') = 0.08 + 0.19 = 0.27$

$$P(E^c) = 1 - P(E) = 1 - 0.73$$

<table>
<thead>
<tr>
<th>S</th>
<th>G</th>
<th>G</th>
<th>G</th>
<th>G</th>
<th>E</th>
<th>F</th>
</tr>
</thead>
<tbody>
<tr>
<td>prob</td>
<td>0.15</td>
<td>0.08</td>
<td>0.21</td>
<td>0.12</td>
<td>0.25</td>
<td>0.19</td>
</tr>
</tbody>
</table>

$E = \{ a, c, d, e \}$ $F = \{b, d, f\}$ $G = \{a, b, d\}$

D) $P(F \cap G) = 0.08 + 0.12$

E) $P(E \cup G) = 0.15 + 0.08 + 0.21 + 0.12 + 0.25$

$$= 1 - 0.19$$

Must Know Probability Formulas:

$$P(E \cup G) = P(E) + P(G) - P(E \cap G)$$

$$P(E) = 1 - P(E^c)$$
Example: It is know from a survey that 29% of the people buy product A, 36% of the people buy product B and 11% buy both products. Find the probability that a person selected at random

A) buys only one of the product.

\[ 0.18 + 0.25 \]

B) doesn’t buy either of the products.

\[ 0.46 \]

Example: This table classifies the English, History, Math, and Poly Sci majors at State U according to their year. (There are no double majors.)

<table>
<thead>
<tr>
<th></th>
<th>Freshmen(F)</th>
<th>Sophomores(Soph)</th>
<th>Juniors(J)</th>
<th>Seniors(Sr)</th>
<th>Totals</th>
</tr>
</thead>
<tbody>
<tr>
<td>English(E)</td>
<td>64</td>
<td>35</td>
<td>31</td>
<td>41</td>
<td>171</td>
</tr>
<tr>
<td>History(H)</td>
<td>55</td>
<td>41</td>
<td>33</td>
<td>52</td>
<td>181</td>
</tr>
<tr>
<td>Math(M)</td>
<td>20</td>
<td>54</td>
<td>38</td>
<td>69</td>
<td>180</td>
</tr>
<tr>
<td>Poly Sci(PS)</td>
<td>70</td>
<td>33</td>
<td>41</td>
<td>37</td>
<td>181</td>
</tr>
<tr>
<td>Totals</td>
<td>218</td>
<td>141</td>
<td>155</td>
<td>199</td>
<td>713</td>
</tr>
</tbody>
</table>

If a student is selected at random, find the probability that

A) The student is a History major and a Sophomore.

\[ \frac{41}{713} \]

B) The student is not a Sophomore and is an English major.

\[ \frac{171 - 35}{713} \]

C) The student is a Math major or is a Senior.

\[ \frac{194 + 180 - 69}{713} = \frac{180 + 41 + 52 + 37}{713} \]
Example: Roll a 4 side die and a 6 sided die. Find the probability of

A) getting a sum greater than 9.

\[
\frac{1}{24}
\]

B) getting a sum of 6 and at least one die comes up a 2.

\[
\frac{2}{24}
\]

C) getting a sum of 6 or at least one die comes up a 2.

\[
\frac{11}{24}
\]