Section 6.4: Permutation and Combinations

Standard Deck of Cards: A deck of cards has 4 suits: diamonds, hearts, clubs, and spades. The suits of diamonds and hearts are both red and the suits of clubs and spades are both black. Each suit has the following denominations: Ace, 2, 3, 4, 5, 6, 7, 8, 9, 10, Jack, Queen, and King. The Jacks, Queens and Kings are also called face cards.

Definition: The number of permutations, \( P(n, r) \), of \( n \) distinct items of which \( r \) objects are chosen to be placed in an ordered setting, i.e. row, list,..., is given by \( P(n, r) = \frac{n!}{(n-r)!} \).

Definition: The number of combinations, \( C(n, r) \), of \( n \) distinct items of which \( r \) objects are chosen to be placed in an unordered setting is given by
\[
C(n, r) = \frac{n!}{(n-r)!r!}
\]

Example: Compute: \( C(10, 3) = 120 \) \( \quad P(10, 3) = 720 \)

<table>
<thead>
<tr>
<th>Combinations</th>
<th>Permutations</th>
<th>Multiplication Principle</th>
</tr>
</thead>
<tbody>
<tr>
<td>No repetitions</td>
<td>No repetitions</td>
<td>Reps. or no reps.</td>
</tr>
<tr>
<td>Order is not important</td>
<td>Order is important</td>
<td>Order is important</td>
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<td>groups</td>
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<tr>
<td>samples</td>
<td>seating charts</td>
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<td>card hands</td>
<td>numbers</td>
<td></td>
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</table>
Example: How many different batting orders are possible for a baseball team that has 15 players?

\[ P(15, 9) \]

Example: How many different ways can 4 books be selected from a pile of 8 different books and arranged on a shelf?

\[ P(8, 4) \]

Example: How many ways can you select 4 books to read next week from a pile of 8 different books?

\[ C(8, 4) \]

Example: Six people are to be selected to attend a conference. They are selected from a group that includes 12 freshmen, 9 sophomores, and 10 juniors. How many ways can this be done if

A) all freshmen are selected?

\[ C(12, 6) \]

B) exactly 2 freshmen and exactly 4 sophomores are selected?

\[ C(12, 2) \cdot C(9, 4) \]

C) all freshmen or all sophomores are selected?

\[ C(12, 6) + C(9, 6) \]

D) all freshmen or sophomores are selected?

\[ C(21, 6) \]
E) exactly 2 sophomores and exactly 3 freshmen are selected?

\[ C(9, 2) \cdot C(12, 3) \cdot C(10, 1) \]

F) exactly 4 freshmen are selected?

\[ C(12, 4) \cdot C(19, 2) \]

G) exactly 2 sophomores are selected?

\[ C(9, 2) \cdot C(22, 4) \]
1) At least 4 freshmen are selected?

$$\begin{align*}
\text{F} & \quad 12 \\
4 & \quad 2 \\
5 & \quad 1 \\
6 & \quad 0 \\
\text{Total} & = 25 \\
\end{align*}$$

$$c(12, 4) \cdot c(19, 2) + c(12, 5) \cdot c(19, 1) + c(12, 6) \cdot c(19, 0)$$

J) at least one freshman is selected?

$$\begin{align*}
\text{F} & \quad 12 \\
1 & \quad 5 \\
2 & \quad 4 \\
3 & \quad 3 \\
4 & \quad 2 \\
5 & \quad 1 \\
\text{Total} - \text{don't want} & = 36 \\
C(31, 6) - c(12, 6) \cdot c(19, 0)
\end{align*}$$

Example: How many ways can you get exactly 4 hearts or exactly 3 spades cards in a 6 card hand?

$$n(A \cup B) = n(A) + n(B) - n(\text{A and B})$$

Hearts other 
Spades star.

$$c(13, 4) \cdot c(39, 2) + c(13, 3) \cdot c(39, 3) = 0$$

Example: From a group of 9 people. How many ways can 2 subcommittees be formed where one has 4 people and the other has 3 people.

can be in both committees
$$C(9, 4) \cdot C(9, 3)$$

can not be in both
$$\begin{align*}
C(9, 4) \cdot C(5, 3) \\
& \quad C(9, 3) \cdot C(6, 4)
\end{align*}$$
Example: 100 students are taking a bus trip. How many different ways can the teacher set up a seating chart for the first bus if the bus holds 30 students?

\( \binom{100}{30} \)

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Distinct Rearrangements

How many distinct rearrangements are there for the letters in these words?

kat

\[ 3 \cdot 2 \cdot 1 = 3! \]

kate

\[ \frac{4 \cdot 3 \cdot 2 \cdot 1}{2!} = \frac{4!}{2!} \]

katee

\[ \frac{5!}{3!} \]

\[ \frac{5!}{1! \cdot 1! \cdot 3!} \]
Example: How many ways can the letters of the word mississippi be rearranged?

\[
\frac{11!}{4! \cdot 4! \cdot 2!}
\]

Example: How many ways can the letters of the word Mathematical be rearranged?

\[
\frac{12!}{3! \cdot 2!}
\]

Example: 7 people are asked to each pick a number from 1 to 20. How many ways can exactly 4 of the people pick a number bigger than 13?
Example: A group has 12 guys and 10 girls. How many pictures are possible that contain 7 people in a row if there are exactly 4 boys in the picture?

\[
\binom{7}{4} \left( \frac{12 \cdot 11 \cdot 10 \cdot 9\cdot 10 \cdot 9 \cdot 8}{6 \cdot 6 \cdot 6 \cdot 5 \cdot 5 \cdot 5} \right)
\]

\[
\frac{7!}{4!3!} \cdot \left( \frac{12 \cdot 11 \cdot 10 \cdot 9\cdot 10 \cdot 9 \cdot 8}{6 \cdot 6 \cdot 6 \cdot 5 \cdot 5 \cdot 5} \right)
\]

\[
\binom{12}{4} \cdot \binom{10}{3} \cdot 7!
\]

Example: Your instructor told you that there will be 3 questions with the answer A, 2 questions with the answer B, 1 question with the answer C and 2 questions with the answer D on the 8 question multiple choice question exam given next week. Using this information, how many ways could you answer the exam?

\[
\frac{8!}{3!2!2!}
\]

\[
\binom{8}{3} \cdot \binom{5}{2} \cdot \binom{3}{1} \cdot \binom{2}{2}
\]