Section 5.1: Simple Interest and Compound Interest

Definition: If the principal, \( P \), is invested for a time period of \( t \) at a simple interest rate of \( r\% \) (for that period) then the interest earned at the end of the time period is given by

\[
I = Prt
\]

The future value, \( A \) or \( F \) of the investment at the end of the period is

\[
A = P + I = P(1 + rt)
\]

Example: You invest $500 at an annual simple interest rate of 4% for 6 years. How much interest did you earn? What is the balance at the end?

\[
I = Prt = 500 \times 0.04 \times 6 = 120
\]

\[
A = 500 + 120 = 620
\]

Example: You invest $1000 at a monthly simple interest rate of 6.5% for 2 years. How much interest did you earn? What is the balance at the end?

\[
I = 1000 \times 0.065 \times 2 \times 12 = 1560
\]

\[
A = 1000 + 1560 = 2560
\]
\[ I = Prt \]

Example: You invest $2000 for 8 months and at the end of this time period you have earned $400 of interest. What is the annual simple interest rate? monthly simple interest rate?

\[
\text{Annual Rate:} \\
400 = 2000 \times r \left( \frac{8}{12} \right) \\
r = \frac{400}{2000 \left( \frac{8}{12} \right)} = .3 \rightarrow 30\% \\
\]

\[
\text{Monthly simple interest rate:} \\
400 = 2000 \times r \left( \frac{8}{12} \right) \\
r = \frac{400}{2000 \left( \frac{8}{12} \right)} = .025 \rightarrow 2.5\% \\
\]

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**Compound Interest**

Definition: Suppose the principal, \( P \), is invested for \( t \) years at an annual interest rate of \( r\% \) and interest is compounded \( m \) times per year. The future amount, \( A \) or \( F \), is given by

\[
A = P(1 + i)^n = P \left(1 + \frac{r}{m}\right)^{mt} \\
\]

\[ i = \frac{r}{m} \quad \text{interest rate per period.} \]

\[ n = mt \quad \text{total # of compoundings.} \]
Example: Find the balance of the account if you invest $600 for 7 years at a nominal rate of 5% compounded

A) annually.
\[ A = 600 \left(1 + \frac{.05}{1}\right)^{1 \times 7} = 844.26 \]

B) semiannually.
\[ A = 600 \left(1 + \frac{.05}{2}\right)^{2 \times 7} = 847.78 \]

C) quarterly.
\[ A = 600 \left(1 + \frac{.05}{4}\right)^{4 \times 7} = 849.60 \]

D) monthly.
\[ A = 600 \left(1 + \frac{.05}{12}\right)^{12 \times 7} = 850.82 \]

E) daily.
\[ A = 600 \left(1 + \frac{.05}{365}\right)^{365 \times 7} = 851.42 \]

Example: You want $2000 in an account at the end of 3 years. If the account gets a nominal rate of 5.75% compounded quarterly, how much do you start the account with?

\[ A = P \left(1 + \frac{r}{n}\right)^{nt} \]

\[ \frac{A}{\left(1 + \frac{r}{n}\right)^{nt}} = P \]

\[ 2000 = P \left(1 + \frac{.0575}{4}\right)^{4(3)} \]

\[ P = \frac{2000}{\left(1 + \frac{.0575}{4}\right)^{12}} = 1685.18 \]
Example: You have the choice of investing money in one of two different accounts. The first account is at Bank A and has a rate of 6.51% compounded semiannually. The second account is at Bank B and has a rate of 6.08% compounded daily. Which account is the better deal?

Bank A:

\[ r_{eff} = 100 \left( 1 + \frac{0.0651}{2} \right)^2 - 100 = 6.61515\% \]

Bank B:

\[ r_{eff} = 6.2681\% \]

Definition: For compound interest, the effective yield, \( r_{eff} \), is given by

\[ r_{eff} = 100 \left( 1 + \frac{x}{y} \right)^y - 100 \]

Example: You invest $2000 in an account that pays interest compounded monthly. What interest rate do you need to have a balance of $5,000 at the end of 3 years.

\[ A = P \left( 1 + \frac{r}{t} \right)^{nt} \]

\[ 5000 = 2000 \left( 1 + \frac{r}{12} \right)^{36} \]

\[ 2.5 = \left( 1 + \frac{r}{12} \right)^{36} \]

\[ (2.5)^{\frac{1}{36}} = 1 + \frac{r}{12} \]

\[ (2.5)^{\frac{1}{36}} - 1 = \frac{r}{12} \]

\[ r = 12 \left( (2.5)^{\frac{1}{36}} - 1 \right) = .30935 \]

\[ r = 30.935\% \]
TVM Solver

The TVM solver that is built function on the TI-83/84 calculators. If you are using the old TI-83 press 2nd \( x^{-1} \) and then press ENTER, otherwise press the [APPS] and the select the Finance application and press enter. Here are the variables that are used in the TVM Solver.

- **N**: \( m \times t \) which is the total number of periods (compoundings) for the life of the account.
- **I%**: The interest rate per year as a percentage.
- **PV**: The present value (starting value) of the account.
- **PMT**: This is the payment that is made each period.
- **FV**: The future value (end value) of the account.
- **P/Y**: The number of payments per year.
- **C/Y**: The number of compoundings per year.

For this class, P/Y and C/Y are equal and PMT:END BEGIN should be set to END.